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A new approach to portfolio management in the Brazilian equity market: does the selection of most efficient assets improve performance?

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Abstract

This paper proposes a new strategy for portfolio management in the Brazilian equity market. It comprises the use of Multifractal Detrended Fluctuation Analysis (MF-DFA), a method from econophysics, as a mechanism to select the most efficient assets before computing portfolios weights, i.e. a novel strategy for portfolio composition based on the more adherent equities to the weak-form of market efficiency. Empirical analysis uses daily prices from equity shares negotiated at the Brazilian stock exchange, B3, to compose minimum variance (MVP) and maximum Sharpe ratio (MSR) portfolios, as investing strategies, using monthly, quarterly and annually rebalancing schemes, as the case of non-rebalancing. Out-of-sample performance is evaluated using return and risk measures and includes comparisons against benchmarks, such as equally weighted portfolios and the IBOVESPA. In addition, empirical experiments also include the performance evaluation of the proposed strategy during the recent COVID-19 pandemic. MF-DFA analysis indicated that a multifractal nature for asset price returns was confirmed in the Brazilian stock market, where the equities showed different levels of efficiency, generally associated with long-term persistence. The strategy for optimizing weights under the consideration of only the most efficient equities resulted in portfolios with considerably lower levels of systemic risk, measured by the corresponding betas, indicating that the lack of efficiency is related to higher sensitivity to macroeconomic and conjuncture changes in the economy. The MVP portfolio produces significantly higher performance than the alternative competitors, both concerning risk and return metrics. Additionally, the minimum risk portfolios showed consistently higher returns than the IBOVESPA, with a level of volatility generally twice smaller than this benchmark. Finally, concerning the analysis during the COVID-19 pandemic, besides the consistent negative impact of the crisis in all portfolios performances, MVP and MSR portfolios composed by the most efficient stocks verified significantly better results (lower losses) than the IBOVESPA.

Keywords: Portfolio selection, Efficiency, MF-DFA, Equity Markets, B3.

Resumo

Este trabalho propõe uma nova estratégia para a gestão de carteiras no mercado acionário brasileiro. Baseia-se no uso de Funções de Flutuações Destendenciadas Multifractais (MF-DFA), um método da econofísica, utilizado para selecionar ativos mais eficientes antes de computar os pesos dos portfolios, ou seja, uma nova estratégia para seleção de carteiras com base nas ações mais aderentes ao conceito de eficiência de mercado, em sua forma fraca. A análise empírica usa preços diários de ações negociadas na B3 para compor carteiras de variância mínima (CVM) e que maximizam a razão de Sharpe (CMS), como estratégias de investimento, sem rebalanceamento e com rebalanceamento mensal, trimestral e anual. O desempenho fora da amostra é avaliado em termo de retorno e risco, e inclui comparações com benchmarks, como carteiras igualmente ponderadas e o IBOVESPA, assim como a avaliação dos desempenhos da estratégia proposta durante a pandemia da COVID-19. O método MF-DFA indicou que os retornos dos preços dos ativos têm dinâmica multifractal, sobretudo com persistência de longo prazo, e as ações apresentaram diferentes níveis de eficiência. A estratégia que considera apenas as ações mais eficientes resultou em carteiras com níveis de risco sistêmico consideravelmente mais baixos, medidos pelos respectivos betas, indicando que a ineficiência está relacionada a maior sensibilidade às mudanças conjunturais da economia. A carteira CVM produziu um desempenho significativamente maior do que os demais métodos em termos de risco e retorno. Além disso, as carteiras de risco mínimo apresentaram retornos consistentemente superiores aos do IBOVESPA, com um nível de volatilidade geralmente duas vezes menor do que este benchmark. Por fim, no que se refere à análise durante a pandemia da COVID-19, além do consistente impacto negativo da crise no desempenho dos portfólios, as carteiras CVM e CMS, compostas pelas ações mais eficientes, verificaram resultados significativamente melhores (menores perdas) que o IBOVESPA.

Palavras-chave: Seleção de carteiras, Eficiência, MF-DFA, Mercado de Ações, B3.

1 Introduction

Since the development of the modern portfolio selection theory, proposed in the seminal paper of Markowitz (1952), the mean-variance approach has played an important role in portfolio management for both academics and market participants. The theory provides a framework to determine optimal portfolio weights associated with the lowest level of risk for a given expected portfolio return. The weights are found as the solution of an optimization problem that requires the estimation on the means and covariances of assets returns (XING; HU; YANG, 2014).

The empirical literature has shown a poor out-of-sample portfolios performance when means and covariances are inaccurate under the methodology of Markowitz (JAGANNATHAN; MA, 2003; MERTON, 1980). To cope with this issue, for instance, Jagannathan and Ma (2003) focused on the estimation of minimum variance portfolios, as the associated optimization problem is reduced to minimize portfolio variance and does not require assets returns mean estimation. Jiang, Du and An (2019), Bodnar, Parolya and Schmid (2018), Bodnar, Mazur and Okhrin (2017), Frahm and Memmel (2010) are exemples of different approaches to compose minimum variance portfolios in equity markets.

A great number of researches also dealt with alternative methods to improve accuracy of assets risk and return estimation in the mean-variance framework, such as fuzzy logic (MASHAYEKHI; OMRANI, 2016; YOSHIDA, 2009), data envelopment analysis (ESSID; GANOUATI; VIGEANT, 2018; LIM; OH; ZHU, 2014), autoregressive and moving average models (PINTO; MONTEIRO; NAKAO, 2011), extreme value volatility models (KARMAKAR, 2017; DIMITRAKOPOULOS; KAVUSSANOS; SPYROU, 2010), Bayesian techniques (BODNAR; MAZUR; OKHRIN, 2017), cointegration and correlation methods for index tracking (SANT'ANNA; FILOMENA; CALDEIRA, 2017), artificial neural networks (YU; WANG; LAI, 2008; FERNÁNDEZ; GÓMEZ, 2007), support vector machines (PAIVA et al., 2019), particle swarm optimization (SILVA; HERTHEL; SUBRAMANIAN, 2019), realized volatility estimators (CALDEIRA et al., 2017), and jump-diffusion processes (LIAN; CHEN, 2019). Generally, the authors stated that these approaches do impact weights estimation errors and therefore the out-of-sample portfolios performances.

Strategies based on momentum of asset returns are also commonly developed in the empirical literature. A plethora of studies provides evidence of positive excess returns for investors who hold positions in stocks with best historical performance combined with short positions in stocks with the worst historical performance (PRATO; WU, 2004; LEWELLEN, 2002; GRUNDY; MARTIN, 2001; CHAND; JEGADEESH; LAKONISHOK, 1996; JAGADEESH; TITMAN, 1993). On the other hand, researches advocated advantages of contrarian strategies as well, i.e. holding assets with worst historical performance and shorting stocks with highest historical performance, which are also able to produce abnormal returns (CHOPRA; LAKONISHOK; RITTER, 1992; RICHARDS, 1997).

To manage and control the volatility in portfolio selection, the financial literature has suggested the use of more sophisticated risk measures such as the Value-at-Risk (VaR). For instance, Jang and Park (2016) presented a model of optimal portfolios integrating a Value-at-Risk constraint. Dynamic portfolio management based on VaR exposure is also addressed in the work of Basak and Shapiro (2001). To overcome the problems associated with the VaR, i.e. lacking of coherence as a risk measure and absence of sub-additivity and convexity, mean-variance approaches using Conditional-VaR (C-VaR) to manage and control portfolios risk are also commonly adopted (XU et al., 2016; BANIHASHEMIA; NAVIDI, 2017).

Alternatively, studies have also investigated the optimal portfolio composition using asset pricing techniques, such as the capital asset pricing model (CAPM) to select the universe of investing alternatives (stocks) before optimization. Bao, Diks and Li (2018) evaluated how specification of the error term affects the CAPM. The authors indicated that the generalized CAPM with identically asymmetric power distributed errors has desirable properties, and showed that portfolios constructed using this alternative outperforms portfolios constructed with normally distributed errors in terms of risk and return. Based on the possible losses by combining bonds and shares corresponding to the CAPM, Spreitzer and Reznik (2007) optimized the composition of a portfolio. They indicated that the suitability of premium definition, i.e. proportional to the loss, does influence portfolio selection and out-sample performance.

Besides the strategy used for setting portfolio weights, most of the decision supporting financial models are constructed under the assumptions of the well-known Efficient Market Hypothesis (EMH), proposed by Fama (1965). EMH stated essentially that an efficient market provides to all participants the same information, and this information is totally reflected into asset prices. This hypothesis has been tested by a plethora of studies in the empirical literature, mostly its weak-form version of market efficiency, i.e. the unpredictability of security returns on the basis of past price changes. The works of Markiel and Fama (1970) and Titan (2015) provide interesting surveys of the existing literature concerning efficiency tests in financial markets. The adherence of asset returns to the EMH are not commonly verified empirically, being the weak-form of efficiency considered too restrictive. Further, mainly for stock markets, the analysis of prices efficiency influences efficient allocation of resources, being crucial for investors, regulators and policy makers, thus, for the financial markets growth and the economy in general.

A recent and novel framework for asset returns efficiency analysis is the use of multifractal detrended fluctuation analysis (MF-DFA). From econophysics, the method provides a tool for examining financial time series stylized facts such as multifractality, asymmetry, persistence and long-memory dependences, which are essential for portfolio management as they influence the predictability of future prices, thus market inefficiency (AL-YAHYAEE et al., 2020). In comparison to rescaled range analysis (R/S), MF-DFA, based on the behaviuor of q-dependent fluctuation functions, may accurately detect the long-range autocorrelations in financial markets, being not prone to time series non-stationarity and short-term autocorrelation issues, as verified in the R/S method, likely to provide spurious results of long-memory parameters (ALI et al., 2018; ZHUANG; WEI; MA, 2015). In addition, another advantage is that MF-DFA provides the construction of a measure to rank the markets based on their efficiency degrees, by evaluating stationarity and random walk properties of asset returns in terms of the spectrum of generalized Hurst exponents.

Hence, this paper aims to investigate how assets efficiency influences optimal portfolio composition in the mean-variance framework for the Brazilian equity market. It is proposed a new strategy for portfolio composition with the use of MF-DFA as a mechanism to select the most efficient assets before optimizing portfolios in Brazil. Portfolios' out-of-sample performances are evaluated under various scenarios concerning investors objectives and rebalancing frequencies. Empirical analysis comprises equity shares negotiated at the Brazilian stock exchange, B3, to compose minimum variance and maximum Sharpe ratio portfolios, as investing strategies, using monthly, quarterly and annually rebalancing schemes, as well as the case of non-rebalancing of the portfolios. The main objective is to test a novel strategy for portfolio management based on the most efficient stocks, i.e. with the most adherent assets to the weak-form of market efficiency, hypothesis commonly assumed by traditional financial models. Out-of-sample performance is evaluated using several return and risk measures and includes comparisons against benchmark investing approaches, such as equally weighted portfolios and the IBOVESPA, the main stock index of the Brazilian equity market. In additional, empirical experiments also include the performance evaluation of the proposed strategy during the recent Corona Virus Disease (COVID-19) pandemic.

The contributions of this paper to the literature and market participants are summarized as follows. First, to the best of our knowledge, it is the first research that uses econophysics MF-DFA analysis as a strategy in portfolio selection in the realm of the mean-variance framework. Hence, this is an innovative quantification model to deal with the trade-off between risk and return in portfolio management, i.e. a new trading strategy that accounts for multifractality in assets returns by measuring its impacts on investment overall performance. Second, it embodies an extensive empirical experiment focusing on how stocks efficiency influences portfolio out-of-sample perfomance under different rebalancing scenarios by achieving new evidences on the relation among means and covariance matrix estimation error and portfolio performance in the Markowitz framework. Third, the suggested approach appears as a feasible, alternative and simple trading strategy which may be useful for market participants to improve their decision making processes for resources allocation. Fourth, experiments concerning actual financial data from an emergent economy like the Brazilian equity market is also a contribution of the work. The B3, the São Paulo stock exchange is among the greatest exchanges in the world with an average daily spot market trading volume greater than BRL 30 billions on Jul. 2020[1], being composed by companies with an average market capitalization over BRL 13 billions[2]. As emerging markets are more prone to market inefficiencies (SÁNCHEZ-GRANERO et al., 2020), accurate techniques for investment decisions are crucial and demanding. Finally, this study is also an attempt to contribute to the literature by exploring the suggested investment strategy on an environment of high levels of systemic risk as during the recent COVID-19 pandemic in order to evaluated whether or not portfolios composed by most efficient equities are less prone to the negative effects of the recent coronavirus instabilities.

The remainder of this work is organized as follows. Section 2 provides a literature review on the use of MF-DFA analysis for accessing assets inefficiencies in the financial markets. The methodology concerning the construction of fluctuation functions to access stocks efficiency, the strategies for portfolio selection adopted, as well as the out-of-sample performance measurements are described in Section 3. Empirical experiments are detailed in Section 4. Finally, Section 5 summarizes the main conclusion and suggests topics for future research.

2 Literature review

The Efficient Market Hypothesis (EMH), since its proposition by Fama (FAMA, 1965; FAMA, 1970), has been widely evaluated in the empirical finance literature, emphasizing the random walk dynamics for asset price returns. Under this hypothesis, future asset returns cannot be predicted based on historical prices information. Hence, a market is considered efficient, in its weak-form, if prices reflect all publicly available historical information (FAMA, 1970). However, the rejection of random walk hypothesis does not necessarily indicate that a market is informational inefficient, justifying the continuous analysis by academics and market participants, given the implications for resources allocation in the capital markets.

Although numerous studies tested the EMH, there is no consensus in the literature,

[1] Source: <http://www.b3.com.br/pt_br/market-data-e-indices/servicos-de-dados/market-data/consultas/mercado-a-vista/dados-de-mercado/>. Access on Aug. 26, 2020.

[2] Source: <http://www.b3.com.br/pt_br/market-data-e-indices/servicos-de-dados/market-data/consultas/mercado-a-vista/valor-de-mercado-das-empresas-listadas/bolsa-de-valores/>. Access on Aug. 26, 2020.

as results depend on the empirical approaches (markets, data frequency, period for evaluation), on the theoretical models considered, as well as on the heterogeneity of inferences (for the same data set, some methods confirm EMH, while others do not). For example, evidence of the weak-form of market efficiency has not been rejected by the literature for stock markets in the following countries: United States (PESARAN; TIMMERMANN, 1995); Canada (ALEXEEV; TAPON, 2011); Bulgaria, Czech Republic, Slovakia and Hungary (HASANOV; OMAY, 2007); United Arab Emirates (MARASHDEH; SHRESTHA, 2008); Iran (OSKOOE; LI; SHAMSAVARI, 2010); India, Pakistan, Bangladesh, and Siri Lanka (SHAHZAD et al., 2018); Indonesia, Malaysia, Singapore and South Korea (RIZVI; ARSHAD, 2014). Anagnostidis, Varsekelis and Emmanouilidesa (2016), concerning indices of different stock markets in Europe for the period from 2004 to 2014, pointed out that mean reversion patterns are observed in the period after the 2008 crisis, whereas previously price variations are more suited to the random walk hypothesis.

On the other hand, studies presented empirical evidences on the rejection of the hypothesis of market efficiency, even in its weak-form. Among these, we can mention the work of Tabak (2003) that considers the Brazilian stock market. The results by Wang, Zhang and Zhang (2015), for example, indicated the rejection of the weak-form of market efficiency for several Asian countries, such as Japan, China, Hong Kong, Malaysia, Singapore, and Thailand. Similar conclusions are also attested for the stock markets in Spain (METGHALCHI; CHEN; HAYES, 2015), Blangladesh (UDDIN; KHODA, 2009), and for the countries of the Gulf Cooperation Council (JAMAANI; ROCA, 2015).

An alternative tool to test the EMH, widely considered recently, consists on the use of multifractal detrended fluctuation analysis (MF-DFA). The works of Cajueiro and Tabak (2004), Cajueiro and Tabak (2005) and Matte, Aste and Dacorogna (2005) are examples on the use of mono-fractal techniques to rank and compare efficiency in capital markets. However, there is an evidence in the literature that attests to the inadequacy of fractal methods with only one exponent of scale (mono-fractals), as they can result in spurious inferences (KWAPIEN; OSWIE; DROZDZ, 2005; PASQUINI; SERVA, 1999). To overcome this limitation, multifractal detrended fluctuation analysis, proposed by Kantelhardt et al. (2002), appears as a flexible and efficient method to test multifractal properties (long-memory) in non-linear time series (MENSI; TIWARI; YOON, 2017).

Multifractal structures in time series are not identified by traditional methodologies, as in nonlinear variance ratio tests such as BDS, or autocorrelation functions, for example. As fractal dynamics are observed in time series associated with heavy tails and long-memory, as in financial asset price returns, econophysics has stated the advantages of evaluating market efficiency for stocks in its weak-form through the use of the MF-DFA technique (ARSHAD; RIZVI; GHANI, 2016; ALI et al., 2018; TIWARI; AYE; GUPTA, 2019).

For the Organization for Islamic Cooperation countries, Arshad, Rizvi and Ghani (2016) used MF-DFA multifractal technique to test the random walk hypothesis in the respective stock markets, i.e. the weak-form of efficiency. The results showed that the markets suffered an increase in information efficiency over time. Similarly, a change in the inefficiency levels for Saudi Arabian banks is stated by Mensi et al. (2018) using MF-DFA and Multifractal Detrended Cross-Correlation Analysis (MF-DXA). Sukpitak and Hengpunya (2016) also observed this relationship in market efficiency based on detrended fluctuation analysis for Thailand in the period from 1975 to 2015.

Using data from US exchange traded funds (ETF) markets, Tiwari, Albulescu and Yoon (2019) evaluated the efficiency of several ETF indices. The authors showed empirical evidence of multifractal nature for the series and that efficiency varies over time, even when it was negatively affected by the mortgage crisis (subprime). The effects of the 2008 crisis on the reduction in efficiency are also attested to the Islamic stock market in Mensi, Tiwari and Yoon (2017). Shahzad et al. (2017) examined the power law properties of 11 US credit and stock markets at the industry level using MF-DFA and discussed the relative efficiency of these markets. They also evaluated the mutual interdependence between Credit Default Swaps (CDS)-equity market pairs. For the period from December 2007 to December 2014, results indicated that: i) CDS markets are relatively more inefficient than their equity counterparts; ii) banks and financial credit markets are relatively more efficient; and iii) basic materials (both CDS and equity indices) is the most inefficient sector of the US economy (SHAHZAD et al., 2017).

Ali et al. (2018) analyzed the stock market efficiency in Islamic countries, such as

Jordan, Malaysia, Pakistan and Turkey using the MF-DFA method. The authors attested the evidence of multifractal structures in stock returns as well as an increase in efficiency over time. Similar evidence was also found by Lin, Fei and Wang (2011) for the Chinese market according to multifractal fluctuation analysis. Combining MF-DFA and multivariate GARCH models, Rizvi and Arshad (2017) showed that there is also a trend towards increasing the efficiency of Japan's stock market from 1990 to 2014.

Tiwari, Aye and Gupta (2019) evaluated the evidence of multifractals and the hypothesis of market efficiency for eight developed countries (Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom and United States) and two emerging ones (India and South Africa). The results indicated that the stock markets are multifractal in nature and have long-term persistence for both developed and emerging economies. In addition, for all cases, the level of efficiency varies over time and, in some cases, the hypothesis of random walk is not rejected.

Recently, Al-Yahyaee et al. (2020) evaluated the efficiency hypothesis in digital coin markets. Based on the application of MF-DFA for Bitcoin, Ethereum, Monero, Dashcoin, Litecoin and Ripple returns, authors stated that the inefficiency of cryptocurrency markets is time-varying, being Dashcoin the least inefficient and Litecoin the most inefficient. In addition, quantile regression analysis provided evidences that high liquidity with low volatility help active traders to arbitrage away opportunities, resulting in market efficiency (AL-YAHYAEE et al., 2020). Similarly, the work of Cheng, Liu and Zhu (2019) indicated that cryptocurrencies have multi-fractal characteristics, implying that market's response to information is non-linear and the investor behavior under different time scales also exhibits a nonlinear state. In addition, Al-Yahyaee, Mensi and Yoon (2018) indicated that multifractality of the Bitcoin market was stronger and Bitcoin was therefore more inefficient than the gold, stock and currency markets using data for the 2010-2017 period.

Hence, a great number of studies have found evidence of multifractal nature in financial asset returns. Therefore, the use of multifractal detrended fluctuation analysis (MF-DFA) to properly measure returns long-range dependences plays a key role in management applications that are influenced by market efficiency, as in portfolio selection, mainly in emergent economies such Brazil, where the presence of inefficiencies and microstructure noises are more relevant.

3 Methodology

The aim of this paper is to evaluate, in the Brazilian equity market, whether the strategy of investing in more efficient stocks does promote better results to investors portfolios' in terms of risk and return. First, based on an universe of possible stocks to invest, they are ranked in terms of efficiency using multifractal detrendeded fluctuation analysis (MF-DFA), accordingly to a measure of market deficiency, calculated by the corresponding generalized Hurst exponents. Second, based on the most efficient stocks, portfolios are constructed under the mean-variance framework of Markowitz along with two different strategies: the minimum variance portfolio and the maximum Sharpe ratio portfolio. Third, portfolios out-of-sample performance is evaluated according to different rebalancing frequencies and against traditional stock market benchmarks. Each step of the methodology is detailed in the following.

3.1 Multifractal detrended fluctuation analysis

The trading strategy proposed in this study departs from an universe of possible stocks to compose efficient portfolios under the Markowitz framework. Equities selection is based on their respective market efficiency, measured in terms of the adherence to the weak-form of efficiency. Multifractal detrended fluctuation analysis (MF-DFA) is used to rank the stocks in terms of efficiency. MF-DFA comprises a tool to measure persistency, anti-persistency, and random walk behavior for a time series through a spectrum of generalized Hurst exponents. In the case of stock returns, the level of assets market inefficiency is ranked based on their relative inefficiencies according to the corresponding q-th order Hurst exponents.

As detailed by Kantelhardt et al. (2002), the method of MF-DFA is comprised by five steps. In the context of this paper, the series under analysis, $\{r(t)\}$, for $t = 1, 2, \ldots, T$, where T is the number of observations, represents the time series of assets log-returns, calculated as $r(t) = \ln[P(t)] - \ln[P(t-1)]$, where $P(t)$ denotes the equity price at time

t. The first step of MF-DFA is to compute the profile function of $r(t)$, denote as $y(t)$, according to:

$$y(t) = \sum_{k=1}^{t} [r(k) - \bar{r}], \tag{1}$$

where $\bar{r}$ is the sample mean of r.

The second step is the division of the profile $y(t)$ into $T_s \equiv \text{int}(T/s)$ non-overlapping segments (windows) of equal length s, where s is known as the scale parameter. The obtained intervals may not include any segment of the time series, as the sample size is not necessarily a multiple of the scale parameter s. Hence, the series $y(t)$ should be divided from the opposite again to make sure no information is lost, generating $2T_s$ sub-intervals (TIWARI; AYE; GUPTA, 2019; BAI; ZHU, 2010).

In each generated sub-interval, $\{\nu = 1, \ldots, 2T_s\}$, the third step consists in the calculation of the local tendency using least squares. Thus, the corresponding detrended time series is obtained by the difference between the actual value and its estimate (tendency):

$$y_s(t) = y[(\nu - T_s)s + 1] - y_\nu(t), \text{ for } \nu = 1, \ldots, T_s, \tag{2}$$

and

$$y_s(t) = y[T - (\nu - T_s)s + 1] - y_\nu(t), \text{ for } \nu = T_s + 1, \ldots, 2T_s, \tag{3}$$

where y_ν denotes the estimated polynomial at the ν-th segment.

The variances are estimated for $\nu = 1, \ldots, T_s$ and $\nu = T_s + 1, \ldots, 2T_s$, respectively:

$$F^2(s, \nu) = \frac{1}{s} \sum_{t=1}^{s} \{y[(\nu - 1)s + t] - y_\nu(t)\}^2, \tag{4}$$

$$F^2(s, \nu) = \frac{1}{s} \sum_{t=1}^{s} \{y[T - (\nu - T_s)s + t] - y_\nu(t)\}^2. \tag{5}$$

In the fourth step, the fluctuation function of order q, $F_q(s)$, is calculated by averaging over all segments (subsets):

$$F_q(s) = \left\{ \frac{1}{2T_s} \sum_{\nu=1}^{2T_s} [F^2(s, \nu)]^{q/2} \right\}^{1/q}. \tag{6}$$

The order q can take any real number except zero. For $q = 0$, $h(0)$ cannot be determined directly because of the diverging exponent. Instead, a logarithmic average procedure has to be employed. For $q = 2$, the standard DFA procedure is retrieved (TIWARI; AYE; GUPTA, 2019).

Finally, the fifth step consists on the analysis of the log-log plots of $F_q(s)$ against s for each value of q, which provides the scaling behavior of the fluctuation functions. If the series $r(t)$ are long-range correlated, $F_q(s)$ increases when s gets larger, i.e. $F_q(s) \sim s^{h(q)}$, known as the power-law.

The Hurst exponent, H ($\equiv h(2)$), is generalized by the scale exponents $h(q)$ and brings information regarding the pattern of a time series (ALI et al., 2018). If $h(q)$ does not depend on q, the time series is monofractal, otherwise, it is multifractal. Moreover, if $0 < h(q) < 0.5$ ($0.5 < h(q) < 1$) the time series has anti-persistence (persistence) dynamics, being this measures, for example, associated with the level of inefficiency of a market. When $h(q) = 0.5$, the stochastic process under study corresponds to an uncorrelated geometric Brownian motion or, in other words, it is an indicative of the non-rejection of market efficiency on its weak-form.

Hence, to determine the level of inefficiency of a market, we considered a market deficiency measure (MDM), calculated as (TIWARI; AYE; GUPTA, 2019):

$$\text{MDM} = \frac{1}{2}\left(|h(q_{min}) - 0.5| + |h(q_{max}) - 0.5|\right), \tag{7}$$

where q_{min} and q_{max} are the minimum and maximum values used on the determination of the fluctuation function in Eq. (6) (TIWARI; AYE; GUPTA, 2019; ALI et al., 2018; MENSI; TIWARI; YOON, 2017).

Under the weak-form of market efficiency (random walk), the function $h(q)$, for different values of q, equals to 0.5. Thus, according to the MDM measure in (7), a market is efficient when MDM is zero, whereas a higher value of MDM indicates a less efficient market. This measure allows the ranking of different stocks in terms of efficiency, therefore, enables the selection of the most efficient assets to compose portfolios, which is the main objective of this work.

3.2 Strategies for portfolio selection

After ranking the equities in terms of efficiency using MDM measure from MF-DFA, the corresponding efficient asset-based portfolios are constructed. To compose these portfolios, the mean-variance framework of Markowitz (1952) is considered under two different investing strategies: the Minimum Variance Portfolio (MVP) and the Maximum Sharpe Ratio Portfolio (MSR), described in the following.

Under an universe of N assets and the corresponding portfolio represented by the vector of weights $\mathbf{w} = (w_1, w_2, \ldots, w_N)^T$, the MVP is obtained as the solution of the following optimization problem:

$$\min_{\mathbf{w}} \left[\sigma_p^2\right] = \min_{\mathbf{w}} \left[\mathbf{w}^T \Sigma \mathbf{w}\right], \text{ subject to } \sum_{i=1}^{N} w_i = 1, \tag{8}$$

where $\sigma_p^2 = \mathbf{w}^T \Sigma \mathbf{w}$ is the portfolio variance, Σ a $N \times N$ matrix of covariances, and the constraint $\sum_{i=1}^{N} w_i = 1$ stands for a full-invested portfolio.

The MVP formulation in (8) subsumes the possibility of short and long positions, as $w_i \in \Re,\ \forall i$. By considering only long positions, the $w_i \geq 0\ \forall i$ constrains must be included. Jagannathan and Ma (2003) stated that long-only portfolios in the MVP formulation, when Σ is estimated by the sample covariance, can be considered as a shrinkage approach, which allows the use of the sample covariance matrix as a method that produces suitable results as when more sophisticated and robust methods for estimating the covariance matrix are considered. In addition, the main advantage of considering the MVP approach for portfolio selection is that the problem in (8) can be solved by only concerning the covariance matrix and does not require returns means estimation, as estimation errors on this former statistic have considerable impact on the portfolio weights as stated by Merton (1980).

Alternatively, the Maximum Sharpe Ratio Portfolio (MSR) is the portfolio in the efficient frontier of Markowitz associated with the maximum Sharpe ratio. The Sharpe ratio of a portfolio, SR_p, is defined as:

$$\mathrm{SR}_p = \frac{\mu_p - r_f}{\sigma_p}, \tag{9}$$

where μ_p is the average rate of return of the portfolio, and r_f the risk-free interest rate.

Theoretically, the MSR portfolio produces the allocation associated with the best risk-return trade-off, when the portfolio risk is measured by its volatility, σ_p. The optimization problem related to the MSR portfolio is:

$$\max_{\mathbf{w}} [\mathrm{SR}_p] = \max_{\mathbf{w}} \frac{\mathbf{w}^T \mu_p}{\sqrt{\mathbf{w}^T \Sigma \mathbf{w}}}, \text{ subject to } \sum_{i=1}^{N} w_i = 1, \tag{10}$$

where $\mu_p = (\mu_1, \mu_2, \ldots, \mu_N)^T$ is the vector of assets mean returns.

Finally, the Equally Weighted Portfolio (EWP) strategy is also considered in this work as a benchmark. For an universe of N stocks, the weights in the EWP are calculated as:

$$w_i = \frac{1}{N}, \ \forall i. \tag{11}$$

For practical reasons, this paper also addressed restrictions to the portfolio constructs in MVP, MSR, and EWP. Long-only portfolios were selected with limit constraints associated to portfolio weights, as:

$$0 \leq w_i \leq 0.15, \ \forall i, \ \sum_{i=1}^{N} w_i = 1. \tag{12}$$

The robustness of the results was evaluated under this parameter setting. Simulations indicated that qualitatively similar results in the Brazilian stock markets are achieved when different values for the upper bound for portfolio weights are set[3]. Hence, a limit of 15% for assets maximum weight appears as a reasonable choice for practical purposes, similarly as in the work of Rubesam and Beltrame (2013).

Additionally, this work does not consider the possibility of short positions, as the results can be affected by market conditions associated with the costs of assets loans, which are relevant in the Brazilian stock market, as well as the availability of these loans. Further, since the aim of this research is the empirical evaluation of a strategy based on selecting the most efficient stocks, allowing for short positions may cause a distortion of the results.

Finally, the covariance matrix is estimated using the most simple methodology, i.e. the sample covariance matrix[4]. It is computed based on the time series of assets returns

[3] These results are not reported here but are available under request to the authors.

[4] More sophisticated methods for covariance matrix estimation may be used, such as EWMA and multivariate GARCH-family models. However, testing different methodologies for covariances in portfolio selection is beyond the main objective of this work.

over a pre-specified period. Thus, $\Sigma = [\sigma_{i,j}]$, where $\sigma_{i,j}$ is the covariance between assets i and j, is estimated as:

$$\hat{\sigma}_{i,j} = \frac{1}{T}\sum_{t=1}^{T}(r_{i,t} - \mu_i)(r_{j,t} - \mu_j), \tag{13}$$

where $r_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$ is the log-return of asset i at t, $P_{i,t}$ the asset price at t, and $\mu_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$ the mean return of asset i.

3.3 Performance assessment and backtesting

To access the performance of the composed portfolios, a backtesting procedure is implemented. First, based on an in-sample database, portfolios weights are computed under the different strategies, i.e. MVP, MSR and EWP. The out-of-sample performance is attained into two scenarios: non-rebalancing and rebalancing. Rebalancing concerns monthly, quarterly and yearly frequencies, where the weights are updated on the new estimated portfolio return and covariance matrix.

Several performance measures are computed, such as: annualized returns, cumulative returns, annualized volatility, average one-day Value-at-Risk (VaR), Shape ratio, Modigliani index, maximum drawdown, average turnover, and CAPM alpha and beta. These metrics are described as follows.

The annualized returns of a portfolio, r_p^A, is used for comparing competing portfolios in terms of profitability, and is calculated as:

$$r_p^A = \left[\prod_{t=1}^{T}(1 + r_{p,t})\right]^{252/T} - 1, \tag{14}$$

where 252 stands for the average number of working days over a year.

The cumulative return, r_p^C, calculates the geometric return over a period of time:

$$r_p^C = \left[\prod_{t=1}^{T}(1 + r_{p,t})\right] - 1. \tag{15}$$

To measure the portfolio risk, the annualized volatility, σ_p^A, is calculated as:

$$\sigma_p^A = \sqrt{252} \cdot \sigma_p = \sqrt{252} \cdot \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{p,t} - \mu_p)^2}, \tag{16}$$

where μ_p is the mean portfolio return.

Another measure of risk, usually used by markets participants, is the Value-at-Risk (VaR). The VaR is defined such that the probability of a loss greater than VaR is (at most) $\gamma\%$. Formally, the VaR is defined as:

$$\mathrm{VaR}_\gamma = \inf\{x \in \Re : \mathrm{Prob}(r_p < x) \leq \gamma\}. \tag{17}$$

The one-day VaR is computed for the portfolios using its parametric approach based on a Gaussian distribution at a $\gamma = 5\%$ confidence level. Hence, the average daily $\mathrm{VaR}_{5\%}$ is calculated as an alternative metric for portfolio risk.

Additionally to the portfolio Sharpe ratio, as defined in Eq. (9), the Modigliani risk-adjusted return metric is also considered. It measures the returns of the portfolio, adjusted to the risk, relative to that of some benchmark. The Modigliani index of a portfolio, MI_p, is computed as:

$$MI_p = (r_p - r_f)\frac{\sigma_b}{\sigma_p} + r_f, \tag{18}$$

where σ_b denotes the volatility of a benchmark, i.e. a market portfolio, the IBOVESPA index in this work.

Portfolios maximum drawdown was also calculated. It measures the largest peak-to-trough decline in the value of a portfolio (before a new peak is achieved). Based on the cumulative returns and the maximum cumulative return to that point of the portfolios, a drawdown is defined as the time when the cumulative returns dips below the maximum cumulative returns. Maximum drawdown of a portfolio, MD, is computed as a percentage of that maximum cumulative return, in effect, measured from peak returns.

To estimate the average cost of rebalancing the portfolios, their average turnover is also obtained. The turnover of a portfolio between two periods of rebalancing is calculated as the absolute difference the assets weights. Finally, CAPM's alpha (α) and beta (β) were calculated as the coefficients of the traditional CAPM regression:

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p(r_{b,t} - r_{f,t}) + \epsilon_t, \quad t = 1, \ldots, T, \tag{19}$$

where ϵ_t is an error term, r_b the return of the benchmark (IBOVESPA), and r_f the risk-free interest rate.

4 Empirical experiments

This work evaluates the performance of portfolios composed by the most efficient stocks traded in the Brazilian equity market, ranked in terms of adherence to the weak-form of the efficiency market hypothesis. Using MF-DFA analysis, assets are ranked in terms of efficiency using a market deficiency measure - MDM in Eq. (7). Thus, MVP and MSR portfolios are optimized concerning only the most efficient stocks. This efficient asset-based strategy is evaluated under scenarios with non-rebalancing and based on monthly, quarterly and yearly rebalancing frequencies. Out-of-sample results are compared against traditional benchmarks, such as the IBOVESPA index and an equally weighted portfolio (EWP). In addition, the last subsection additionally provides an evaluation of the suggested approach during the recent COVID-19 pandemic.

4.1 Data

The database is composed by daily closing prices of all equities negotiated at the B3, the Brazilian stock exchange, for the period from Jan. 4, 2010 to Jul. 17, 2020 within a total of 2,612 observations[5]. The sample includes only the assets that presented daily prices and a positive traded volume for all the days in the period evaluated, including both common and preferred shares of a same company, resulting in a sample with 158 assets. In addition, daily series of IBOVESPA index and the interbank interest rate (CDI - Certificados de Depósitos Interfinanceiros) were also collected for the same time span. The CDI rate is used as a proxy for the risk-free interest rate and IBOVESPA as the benchmark market portfolio[6], commonly approach in the Brazilian financial market. Notice that the data corresponding to the year of 2020 are considered only in the particular analysis concerning the COVID-19 pandemic, provided in subsection 4.7. For the following analyses, data are considered from Jan. 4, 2010 to Dec. 30, 2019.

[5] Data were collected at Economatica.

[6] Besides the lack of evidence for IBOVESPA to be considered as an efficient or diversified portfolio, it is consistently used as benchmark in the Brazilian equity market by researches and market practitioners.

4.2 MF-DFA analysis for B3 stocks

The data were divided into in-sample and out-of-sample sets. The in-sample data are comprised by the first three years of daily price returns, from Jan. 4, 2010 to Dec. 28, 2012 - 751 daily observations. This sample is used to perform the ranking of equities in terms of efficiency, as well as for estimating portfolios return and covariance matrix to determine the corresponding weights and to evaluated their performance at the out-of-sample period[7].

The first step of the suggested strategy for portfolio selection is the identification of the most efficient stocks traded at B3. Table 1 shows the main statistics of the generalized Hurst exponents for the 158 equities considered in this paper. In general, the generalized Hurst exponents, $h(q)$, change moderately with a variation in q, that moves from the large and small price fluctuations, denoted by scale exponents $h(-4)$ to $h(4)$, i.e. the generalized Hurst exponents for short- and long-term, respectively. This pattern is an evidence of mutifractality of assets returns as $h(q)$ varies with changes in q, even for the median percentile of the sample, corresponding to the assets with the lowest variations in $H(q)$ to different values of the scale exponent q (see Table 1). Overall, a stronger evidence of long-term persistent is found for the assets traded at B3 ($h(q) > 0.5$), which violates the weak-form of efficiency, except for the 5th percentile that shows evidence of mean reverting ($h(q) < 0.5$), indicating that future returns of the corresponding assets have the tendency to return to a long-term mean for the associated equities.

The fractality of the stock returns is obtained from a log-log plot between the length scale s and the order of fluctuation function - $F_q(s)$ in Eq. (6). To illustrate this pattern, the scaling behavior of the stocks associated with the lower and higher variations in their respective Hurst exponents (slopes $= h(q)$), are shown in Figure 1. The CPFL Energia (ticker CPFE3), a company related to the electric sector in Brazil, showed a lower variation on its generalized Hurst exponents, $h(q)$, in relation to different values in q, i.e. it is considered the most efficient equity at the sample considered. On the other hand, the COMGAS CIA Gás de São Paulo (ticker CGAS3), related to the gas activity in Brazil, showed the higher variation in the generalized Hurst exponents, being associated as the

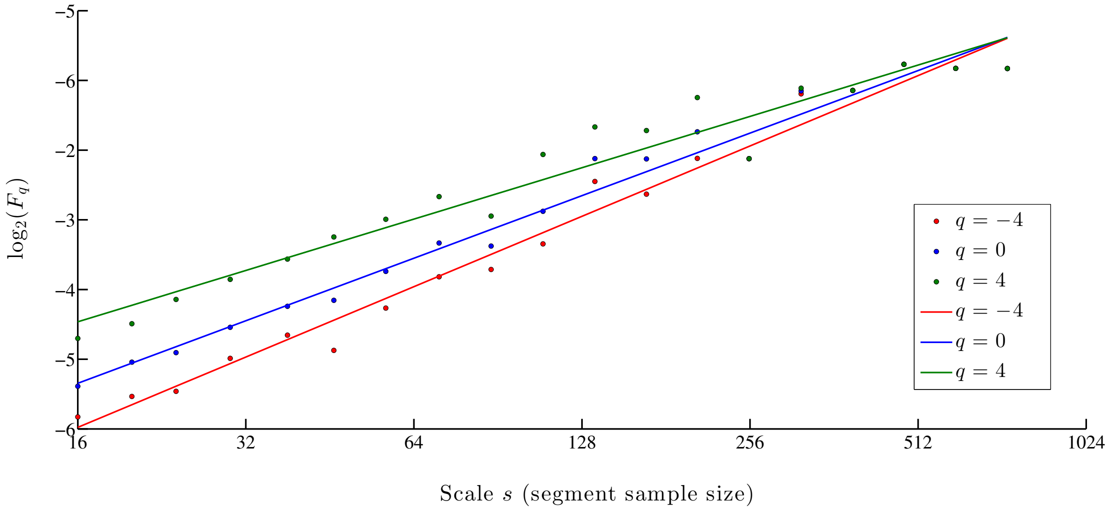
[7] All the analyses developed in this work were implemented in MatLab programming language.

least efficient stock according to MF-DFA analysis. In both cases, it can be observed that the local slope of the plots changes with crossover time scales, which is an evidence of mutifractality, even for the most efficient equity (CPFE3) - see Figure 1.

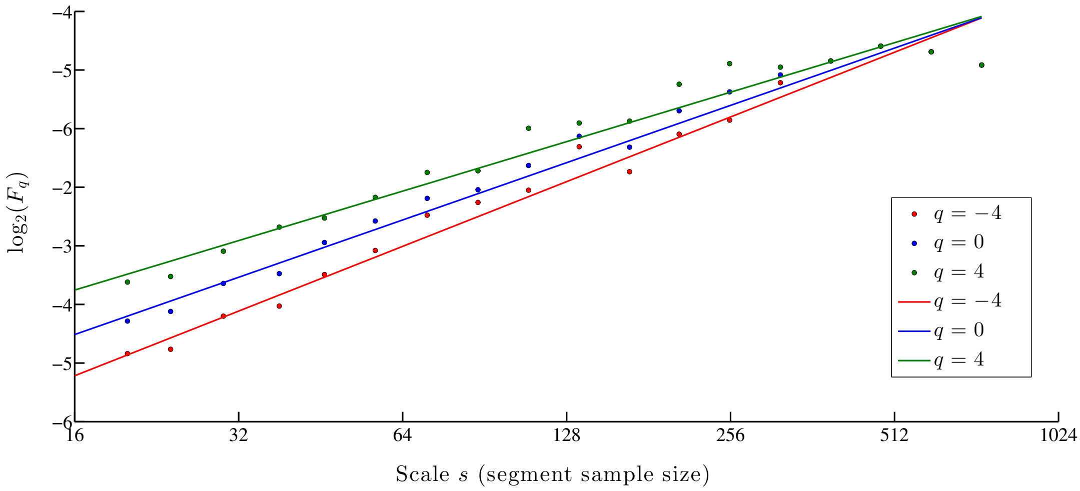
Table 1 – Generalized Hurst exponents statistics for short- and long-term components, from q equals to -4 to 4, calculated for asset price returns of 158 stocks traded at B3 during the period from Jan. 4, 2010 to Dec. 28, 2012.

$H(q)$	q								
	-4	-3	-2	-1	0	1	2	3	4
Average	0.6724	0.6423	0.6047	0.5552	0.4984	0.4416	0.3874	0.3361	0.3039
Std. Dev.	0.1648	0.1628	0.1570	0.1283	0.0762	0.0663	0.0800	0.0999	0.1076
5th Percentile	0.5208	0.4990	0.4725	0.4399	0.4020	0.3347	0.2307	0.1287	0.0804
25th Percentile	0.5796	0.5555	0.5236	0.4848	0.4452	0.3988	0.3518	0.3023	0.2671
Median	0.6535	0.6265	0.5923	0.5513	0.5012	0.4379	0.3886	0.3471	0.3198
75th Percentile	0.7299	0.6965	0.6485	0.5979	0.5394	0.4829	0.4392	0.3999	0.3692
95th Percentile	0.8606	0.8211	0.7632	0.6774	0.6013	0.5530	0.5044	0.4619	0.4329

Due to the evidence of multifractality in the Brazilian equity market, the whole sample was ranked in terms of efficiency using the market deficiency measure (MDM) - Eq. (7). A market is said to be efficient (weak-form) if the value of MDM is close to zero, therefore, a large MDM value indicates a less efficient market. Figure 2 presents the histogram of the MDM values calculated for the 158 stocks negotiated at B3, computed using returns from the in-sample set (Jan. 4, 2010 to Dec. 28, 2012). Notice that the level of efficiency considerably differs among the stocks traded at B3, and the right tail side of the distribution reveals a significant presence of less efficient equities in terms of the weak-form of market efficiency. Hence, taking into account the efficiency of the stocks to compose portfolios may play an important role for asset management in the Brazilian financial market.



(a) CPFL Energia (CPFE3)



(b) COMGAS (CGAS3)

Figure 1 – Scaling function $F_q(s)$ of CPFE3 and TAEE11. The scale s indicates the days. A time series is long-range power-law multifractality correlated if the $F_q(s)$ increases with the scale s.

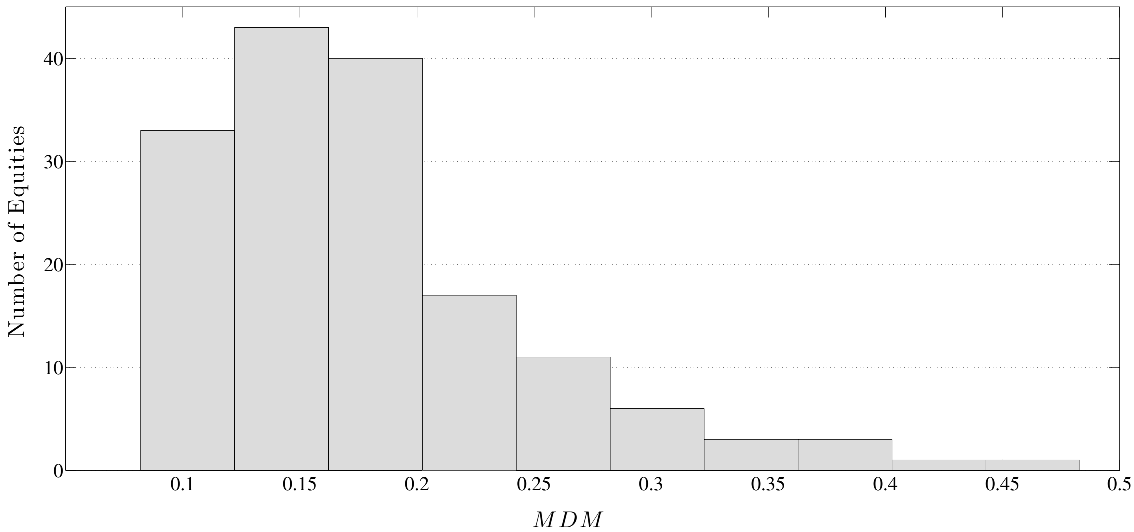


Figure 2 – Histogram of the market deficiency measure (MDM) calculated for the 158 equities traded at B3 using daily price returns for the period from Jan. 4, 2010 to Dec. 28, 2012.

Based on the MDM measure, stocks were ranked in terms of efficiency to compose the corresponding portfolios[8]. Minimum Variance (MVP) and Maximum Sharpe Ratio (MSR) portfolios were optimized in three different cases: concerning an universe of available assets to trade in the most 60, 90 and 120 efficient stocks. The distinct number of stocks considered to determine portfolio weights concerns a mechanism to provide the inclusion of the benefits of diversification, as well as to test the robustness of the results when the number of possible efficient equities to invest varies. The following sections show the performance evaluation of the efficient assets-based portfolios for different scenarios of rebalancing. Subsection 4.7 performs the analysis particularly during the period of COVID-19 pandemic.

4.3 Non-rebalanced portfolios

Table 2 provides the performance evaluation of each portfolio concerning different metrics of risk and return, as: annualized returns (Ann. Ret.), cumulative returns (Cum. Ret.), annualized volatility (Ann. Vol.), average one-day Value-at-Risk at a 5% significance level (Avg. daily $VaR_{5\%}$), the terminal value of investing \$ 1 in each portfolio (Term.

[8] The Table A.1 in Appendix A presents the ranking of all 158 assets considered in this work according to the MDM metric for ranking efficiency.

Value), Sharpe ratio (SR) and Modigliani index (MI), both computed in annualized terms, maximum drawdown (MD), the correlation to the IBOVESPA index (corr. IBOV), CAPM's alpha and beta (α, β), as well as the number of equities in each portfolio (No. Assets) and the corresponding average weights (Avg. Allocation). With non-rebalancing, portfolios were composed based on the in-sample data and the obtained weights are kept the same during the remaining out-of-sample period, i.e. from Jan. 2013 to Dec. 2019.

In terms of returns, results from Table 2 showed that minimum variance portfolios provided higher returns than maximum Sharpe ratio and equally weighted portfolios, independently when a different number of efficient stocks are available to trade in and optimize the weights. Compared to the Brazilian stock market benchmark, the IBOVESPA index, MVP resulted in higher returns when the most efficient 90 and 120 stocks are used to obtain the weights. MVP using the most 90 efficient equities has a higher level of returns, with a corresponding annualized return of 9.97%, even higher than the risk-free interest rate, the CDI, with an annualized return of 9.22% (Table 2). Incorporating the risk in the evaluation, MVP portfolios are also the ones associated with the lowest annualized volatility and average daily VaR, which was expected, as they correspond to the compositions related to the lower level of risk on the efficient frontier of Markowitz. A lower level of risk was also verified for the MSR portfolio composed on an universe of the most 120 efficient stocks, however, it is associated with the worst level of return for MSR strategy. EWP is the strategy associated with the worst results.

Portfolios performances must be evaluated simultaneously in terms of risk and return, as provided by analysing their Sharpe ratio (SR) and Modigliani index (MI). As the capital markets in Brazil is characterized by high levels of interest rates when compared to developed economies, generally SR assumes negative values, which compromises the properly evaluation, and being suitable the consideration of the Modigliani index (Table 2). MVP_{60} presented the highest MI among the competitors, 10,35%, indicating this approach as the best strategy in terms of the risk-return trade-off. Indeed, it was the unique strategy that showed a positive Sharpe ratio among the remaining portfolios. MVP results, for all cases, are associated with the highest values of MI.

Table 2 – Portfolios performance metrics with non-rebalancing. The results are calculated for the out-of-sample set, from the period from Jan. 2013 to Dec. 2019. MVP, MSR and EWP stand for the minimum variance, maximum Sharpe ratio and equally weighted portfolios. IBOV is the IBOVESPA index and CDI the risk-free interest rate. MVP and MSR are optimized considering the most 60, 90 and 120 efficient stocks, ranked using MF-DFA for the returns data from the in-sample set, from Jan. 2010 to Dec. 2012. Term. Value is the terminal value of investing \$ 1 in each portfolio during the period evaluated. No. Assets is the number of assets in each portfolio. Avg. Allocation is the average portfolio weights.

Strategies	MVP			MSR			EWP	IBOV	CDI
	60	90	120	60	90	120			
Ann. Ret.	7.27%	**9.97%**	8.87%	2.48%	5.87%	2.16%	1.23%	7.49%	9.22%
Cum. Ret.	61.88%	**91.99%**	79.01%	18.32%	47.98%	15.82%	8.77%	64.15%	83.16%
Ann. Vol.	15.12%	14.82%	**13.95%**	16.81%	16.38%	14.31%	16.97%	22.36%	0.16%
Avg. daily $\text{VaR}_{5\%}$	-1.52%	-1.49%	**-1.40%**	-1.73%	-1.67%	-1.47%	-1.74%	-2.28%	-
Term. Value	R\$ 1.62	**R\$ 1.92**	R\$ 1.79	R\$ 1.18	R\$ 1.48	R\$ 1.16	R\$ 1.09	R\$ 1.64	R\$ 1.83
SR	-11.80%	**4.63%**	-2.40%	-36.70%	-18.69%	-45.16%	-43.09%	-7.09%	-
MI	6.34%	**10.35%**	8.63%	0.26%	4.66%	-1.81%	-1.30%	-	-
MD	37.01%	**33.24%**	32.58%	41.90%	40.59%	36.97%	59.81%	45.30%	-
IBOV corr.	1.12%	1.44%	1.40%	0.94%	1.18%	0.99%	0.64%	-	-
CAPM β	0.0076	0.0097	0.0089	0.0068	0.0086	0.0059	0.0045	-	-
CAPM α	0.0000	0.0001	0.0000	-0.0002	-0.0001	-0.0002	-0.0002	-	-
No. Assets	21	31	34	15	16	21	-	-	-
Avg. Allocation	4.76%	3.23%	2.94%	6.67%	6.25%	4.76%	-	-	-

Besides MVP portfolios showed similar returns to the IBOVESPA, the Brazilian stock market benchmark is considerably riskier, with an annualized volatility that is approximately twice than the MVP portfolios (see Table 2). This empirical evidence indicates a paradoxal issue to the financial theory, i.e. MVP produces risk-adjusted returns that are superior than other portfolios based on the Markowitz mean-variance framework, as also evidenced in the works of Jagannathan and Ma (2003), Jorion (1991) for the US market, and Thomé, Leal and Almeida (2011) and Rubesam and Beltrame (2013) for the Brazilian case. This evidence has important relevance for academics and market partitioners in the Brazilian equity market. First, it suggests that a simple approach, MVP, as it only requires the covariance matrix estimation to compute portfolio weights, is able to produce higher risk-adjusted returns than more sophisticated methodologies, such as MSR, and even higher when compared to the main benchmark in the Brazilian stock market, the IBOVESPA index, which is quite commonly used for driving asset management strategies in several financial institutions.

Further, one interesting result from Table 2 is the MVP and MSR corresponding CAPM betas, that are generally low. As CAPM beta is associated as a measure of portfolios risk, the systemic risk related to the market benchmark, the IBOVESPA, it is verified that portfolios composed by the most efficient stocks produced lower systemic risks. Notice that, even though CAPM assumes the market benchmark as a diversified portfolio, which is not necessarily the case of the IBOVESPA, when comparing MVP and MSR annualized volatilities, they are generally lower than the IBOVESPA volatility - as also verified by portfolios return correlations to IBOV. In addition, MVP invested in an average of 29 stocks, with a corresponding allocation of 3.64% in each equity (Table 2).

Figure 3 shows the evolution of portfolios cumulative returns, illustrated by the terminal value temporal behavior of investing $ 1 in each strategy at the beginning of the out-of-sample period. The curves related to MVP and MSR concern the cases with higher risk-adjusted return ratio, measured by the Modigliani index (see Table 2), i.e. using the most 90 efficient stocks to compute portfolios weights. For all portfolios, the period from Jan. 2013 to the end of 2016 is associated with losses (terminal value is under $ 1). A reverting performance is observed after the year of 2017, when the cumulative returns observed a significant increase until the end of the sample (Dec. 2019). The best portfolios, with higher cumulative returns, are MVP and the IBOVESPA. However, MVP, for almost the entire period, shows superior cumulative returns than the benchmark (IBOV), being the only portfolio able to produce a terminal value higher that the risk-free interest rate[9]. This performance reveals the high potential of the proposed investing strategy in the Brazilian equity market based on the most efficient stocks, when associated with a minimum risk target (MVP), which is a simple framework for portfolio selection.

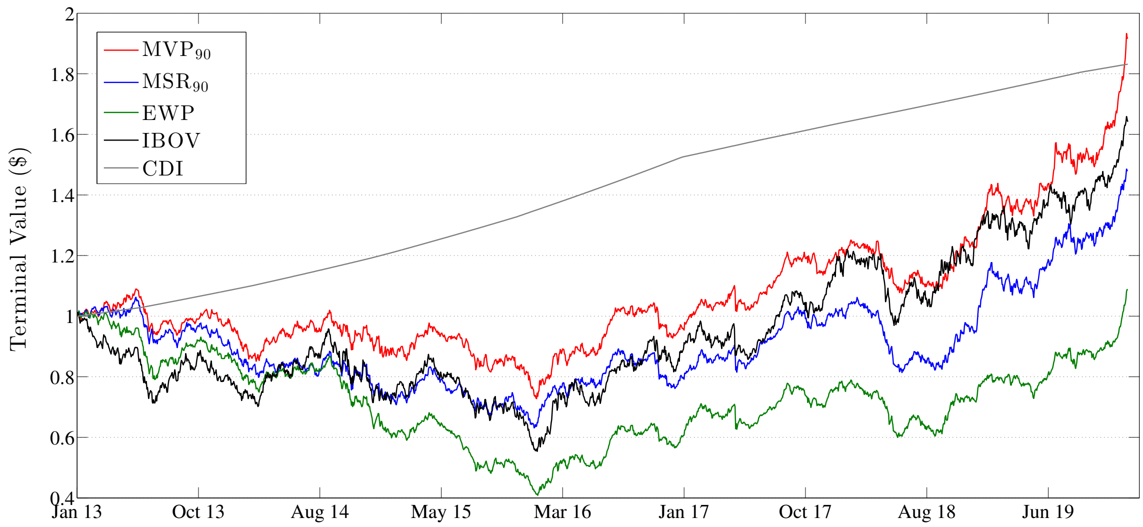


Figure 3 – Terminal value curves of investing $ 1 in each portfolio, as an illustration of cumulative portfolios returns. MVP and MSR curves correspond to the strategies associated with the higher adjusted risk-return relation, measured by the Modigliani index, i.e. considering the 90 most efficient equities as an universe to trade in for both cases.

4.4 Yearly-rebalanced portfolios

Table 3 shows portfolios performance when MVP and MSR are yearly rebalanced. MSR portfolios provided lower returns than the benchmark, the IBOVESPA index, and the minimum variance portfolios. MVP, when the weights are yearly updated, showed

[9] Besides CDI higher returns, it does not necessarily indicate that investors should consider only fixed-income positions to trade in as the best approach. This is because CDI, as an average of interbank loans rates, is generally accessible to investors with a higher capital availability, mainly banks and financial institutions. Thus, for average investors, fixed-income interest rates are computed as a percentage of the CDI rate (around 70% to 90%), depending on the volume and maturity of their operations, which motivates the investment in the stock markets.

less sensible results to the number of the most efficient assets used to obtain portfolio weights during optimization, i.e. for the 60, 90, 120 equities related to the highest market efficiency. Compared to the IBOVESPA, the minimum variance approach overperformed the Brazilian stock benchmark in terms of annualized and cumulative returns, with lower levels of risks, as evidenced by the corresponding annualized volatility and average daily VaR (see Table 3). MVP is also related to the lowest levels of losses during the period, as indicated by the maximum drawdown measure. Notice that the risk-free interest rate, CDI, showed higher return than any of the remaining strategies.

The risk-adjusted return measure, the Modigliani index, suggested that the minimum variance portfolio, optimized considering the most 60 efficient stocks negotiated at B3 is the best strategy for portfolio selection in terms of the return-risk trade-off for yearly rebalancing (Table 3). MVP_{60} was rebalanced seven times and is composed, on average, by 22 assets. The corresponding average turnover was approximately 19%, i.e. 19% of the allocations changed at each time of rebalancing. With rebalancing, transactions costs affect the results of the portfolios, differently from the previous subsection results, as the selected portfolios were kept for the out-of-sample period (non-rebalanced). However, the IBOVESPA index also changes its composition through the time, also implying in transactions costs, reducing the profitability of strategies that try to replicate the stock market benchmark performance.

As portfolios are rebalanced yearly, their returns showed a slightly higher correlation to the IBOVESPA index, in comparison to the case of non-rebalancing. As previously observed, Table 3 indicates that minimum variance approach is able to produce considered lower level of systemic risk than the IBOVESPA, as observed by the lower values of its CAPM betas.

Table 3 – Portfolios performance metrics with yearly rebalancing. The results are calculated for the out-of-sample set, from the period from Jan. 2013 to Dec. 2019. MVP, MSR and EWP stand for the minimum variance, maximum Sharpe ratio and equally weighted portfolios. IBOV is the IBOVESPA index and CDI the risk-free interest rate. MVP and MSR are optimized considering the most 60, 90 and 120 efficient stocks, ranked using MF-DFA for the data from the in-sample set, from Jan. 2010 to Dec. 2012. Term. Value is the terminal value of investing \$ 1 in each portfolio during the period evaluated. Avg. Turnover indicates the average turnover of each portfolio calculated by the absolute difference among portfolios weights within two periods of rebalancing. Avg. No. Assets is the average number of equities in each portfolio.

Strategies	MVP			MSR			EWP	IBOV	CDI
	60	90	120	60	90	120			
Ann. Ret.	**9.04%**	8.89%	7.99%	4.03%	4.14%	7.29%	1.23%	7.49%	9.22%
Cum. Ret.	**81.11%**	79.40%	69.49%	31.16%	32.07%	62.09%	8.77%	64.15%	83.16%
Ann. Vol.	14.40%	13.83%	**13.08%**	15.35%	14.81%	14.41%	16.97%	22.36%	0.16%
Avg. daily $\text{VaR}_{5\%}$	-1.45%	-1.40%	-1.30%	-1.57%	-1.51%	-1.46%	-1.74%	-2.28%	-
Term. Value	**R\$ 1.81**	R\$ 1.79	R\$ 1.69	R\$ 1.31	R\$ 1.32	R\$ 1.62	R\$ 1.09	R\$ 1.64	R\$ 1.83
SR	-1.15%	-2.19%	-8.60%	-30.95%	-31.43%	-12.26%	-43.09%	-7.09%	-
MI	**8.94%**	8.68%	7.12%	1.66%	1.55%	6.22%	-1.30%	-	-
MD	37.68%	36.73%	36.10%	37.25%	37.34%	**35.33%**	59.81%	45.30%	-
IBOV corr.	1.27%	0.86%	0.97%	1.41%	0.64%	1.06%	0.64%	-	-
CAPM β	0.0083	0.0054	0.0057	0.0095	0.0040	0.0068	0.0045	-	-
CAPM α	0.0000	0.0000	0.0000	-0.0001	-0.0001	0.0000	-0.0002	-	-
Avg. Turnover	19.41%	23.70%	23.99%	39.67%	54.25%	89.70%	-	-	-
Avg. No. Assets	22	31	38	13	15	26	-	-	-

Portfolios returns temporal evolution during the out-of-sample period are illustrated in Figure 4 by their terminal value of investing $ 1 in each strategy. MVP and MSR terminal values corresponds to the cases when the 60 and 120 most efficient equities were used to determine the corresponding weights, respectively, as they showed higher risk-adjusted returns relations (MI measure in Table 3). For the whole sample, MVP consistently provides higher returns than the IBOVESPA, except for the period around Jan. 2017, when IBOVESPA provided a slightly better performance. Particularly, during the year of 2017, MSR portfolio appears as the best strategy than MVP, IBOV and EWP. Comparing the cumulative return curves of MVP and IBOV, it is noticed that minimum variance selection approach is less susceptible to returns dropdowns, confirming its lowers CAPM beta and maximum drawdown, as seen in Table 3.

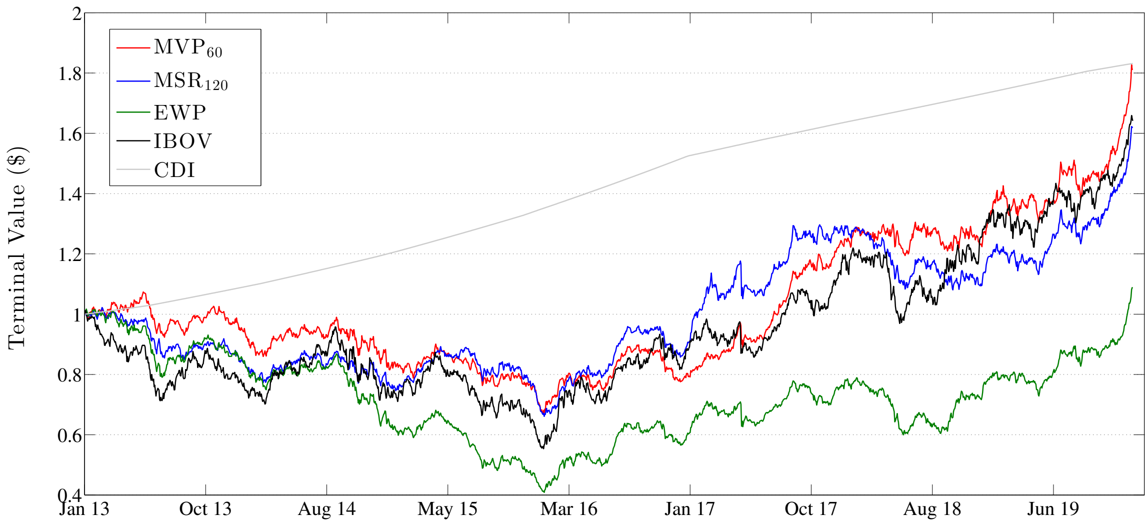


Figure 4 – Terminal value curves of investing $ 1 in each portfolio, as an illustration of cumulative portfolios returns. MVP and MSR curves correspond to the strategies associated with the higher adjusted risk-return relation, measured by the Modigliani index, i.e. considering the 60 and 120 most efficient equities as an universe to trade in, respectively. MVP and MSR were yearly rebalanced.

4.5 Quarterly-rebalanced portfolios

Table 4 summarizes portfolios performances when rebalancing is at a quarterly frequency. For the total out-of-sample set period, 28 rebalances were performed. In terms of return, MVP is the strategy, again, associated with the best performance by means of higher annualized and cumulative returns than MSR, EWP and the IBOVESPA index, except when minimum variance portfolio is optimized under an universe of the 120 efficient stocks traded at B3, where IBOVESPA showed slightly higher returns. In terms of risk, MVP and MSR have similar levels of annualized volatility, but it is considerably lower than the IBOVESPA (Table 4).

Concerning the level of return by unit of risk, the Modigliani indices (MI) from the MVP portfolios are the highest off all remaining investing approaches (Table 4), indicating that a “defencing” strategy (MVP) composed by the most efficient stocks in the Brazilian market do provides better results in terms of the risk-return trade-off under the mean-variance framework of Markowitz. The best MVP portfolio, based on the MI measure, is when it is constructed considering the most 60 efficient stocks to obtain portfolios weights, investing in 22 assets on average, with a mean turnover of 6.82% (see Table 4). As previously verified, both MVP and MSR, using an efficient stocks-based strategy to optimize their weights, provide lower systematic risk, measured by the CAPM betas[10]. Further, it is important to note that, for all cases, CAPM alphas are close to zero, indicated that portfolios returns can be described in relation to the premium returns from the market portfolio IBOVESPA (Table 4).

Figure 5 provides the evolution of cumulative return by investing $ 1 on each portfolio. MVP and MSR were rebalanced quarterly and correspond to their best composition in terms of risk and return, i.e. optimized under the consideration of the 60 and 120 most efficient equities negotiated at B3. Compared to the market benchmark, the IBOVESPA index, it interesting to note that, in most of the evaluated period, MVP generally outperformed its competitors. During almost the entire year of 2017, MSR surprisingly showed the best performance than all alternative methodologies (Figura 5). When MVP is rebalanced at each quarter, compared to previous results (non-rebalancing and yearly rebalancing), this approach sowed a more similar temporal behavior to the IBOVESPA.

[10] For all cases evaluated in this paper, CAPM betas are statistically different from zero at a 1% significance level, and CAPM alphas are statistically insignificant at a 1% significance level.

Table 4 – Portfolios performance metrics with quarterly rebalancing. The results are calculated for the out-of-sample set, from the period from Jan. 2013 to Dec. 2019. MVP, MSR and EWP stand for the minimum variance, maximum Sharpe ratio and equally weighted portfolios. IBOV is the IBOVESPA index and CDI the risk-free interest rate. MVP and MSR are optimized considering the most 60, 90 and 120 efficient stocks, ranked using MF-DFA for the returns data from the in-sample set, from Jan. 2010 to Dec. 2012. Term. Value is the terminal value of investing $ 1 in each portfolio during the period evaluated. Avg. Turnover indicates the average turnover of each portfolio calculated by the absolute difference among portfolios weights within two periods of rebalancing. Avg. No. Assets is the average number of equities in each portfolio.

Strategies	MVP			MSR			EWP	IBOV	CDI
	60	90	120	60	90	120			
Ann. Ret.	**8.34%**	8.26%	7.11%	3.55%	4.34%	6.58%	1.23%	7.49%	9.22%
Cum. Ret.	**73.36%**	72.48%	60.29%	27.02%	33.84%	54.91%	8.77%	64.15%	83.16%
Ann. Vol.	14.00%	13.38%	**12.44%**	15.29%	14.60%	14.06%	16.97%	22.36%	0.16%
Avg. daily $\text{VaR}_{5\%}$	-1.41%	-1.35%	**-1.26%**	-1.57%	-1.49%	-1.43%	-1.74%	-2.28%	-
Term. Value	**R$ 1.73**	R$ 1.72	R$ 1.60	R$ 1.27	R$ 1.34	R$ 1.55	R$ 1.09	R$ 1.64	R$ 1.83
SR	-5.72%	-6.53%	-15.48%	-33.99%	-30.61%	-17.16%	-43.09%	-7.09%	-
MI	**7.82%**	7.63%	5.44%	0.92%	1.75%	5.03%	-1.30%	-	-
MD	37.67%	37.22%	35.24%	37.43%	34.63%	**31.76%**	59.81%	45.30%	-
IBOV corr.	1.46%	1.05%	0.73%	0.67%	0.46%	2.16%	0.64%	-	-
CAPM β	0.0092	0.0063	0.0041	0.0043	0.0028	0.0135	0.0045	-	-
CAPM α	0.0000	0.0000	0.0000	-0.0002	-0.0001	-0.0001	-0.0002	-	-
Avg. Turnover	6.82%	8.79%	9.37%	21.39%	25.79%	48.16%	-	-	-
Avg. No. Assets	22	32	38	12	14	20	-	-	-

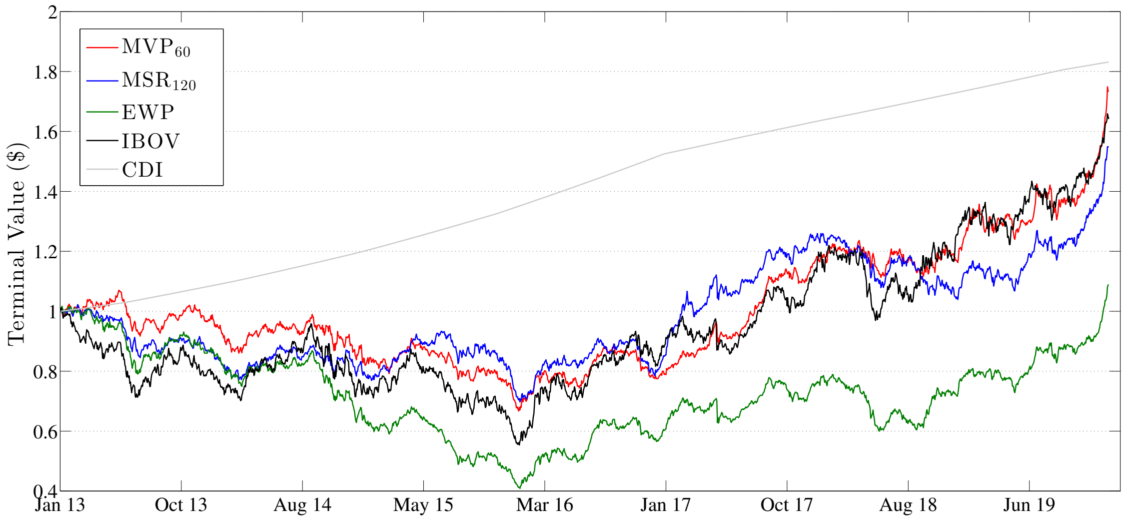


Figure 5 – Terminal value curves of investing $ 1 in each portfolio, as an illustration of cumulative portfolios returns. MVP and MSR curves correspond to the strategies associated with the higher adjusted risk-return relation, measured by the Modigliani index, i.e. considering the 60 and 120 most efficient equities as an universe to trade in, respectively. MVP and MSR were quarterly rebalanced.

4.6 Monthly-rebalanced portfolios

Portfolios performance when rebalancing is performed monthly are reported in Table 5. Results are quite similar to the ones when quarterly rebalancing is considered. Generally, minimum variance portfolios achieved higher returns and lower volatilities when compared against MSR and IBOVESPA for the three cases considered: when the most 60, 90 and 120 efficient stocks are used for optmizing the weights. The annualized volatility of MVP is 13% on average, significantly lower than the Brazilian market benchmark, the IBOVESPA, with a corresponding annualized volatility of 22.36% (see Table 5).

Similarly as previously observed, the strategy based on considering the most efficient stocks, in terms of the weak-form of the efficient market hypothesis, has provided higher risk-adjusted returns, mainly for the minimum risk approach (MVP) based on the 60 most efficient equities, with a corresponding Modigliani index of 7.59% in annualized terms, as shown in Table 5. MVP invested in 30 assets on average and, the best case, MVP_{60}, showed a mean turnover of 3.24%, implying in reduced transaction costs. However, portfolios were monthly rebalanced, within a total of 84 rebalances in the out-of-sample set. Finally, CAPM betas are generally lower for MVP and MSR (Table 5).

Table 5 – Portfolios performance metrics with monthly rebalancing. The results are calculated for the out-of-sample set, from the period from Jan. 2013 to Dec. 2019. MVP, MSR and EWP stand for the minimum variance, maximum Sharpe ratio and equally weighted portfolios. IBOV is the IBOVESPA index and CDI the risk-free interest rate. MVP and MSR are optimized considering the most 60, 90 and 120 efficient stocks, ranked using MF-DFA for the returns data from the in-sample set, from Jan. 2010 to Dec. 2012. Term. Value is the terminal value of investing $ 1 in each portfolio during the period evaluated. Avg. Turnover indicates the average turnover of each portfolio calculated by the absolute difference among portfolios weights within two periods of rebalancing. Avg. No. Assets is the average number of equities in each portfolio.

Strategies	MVP			MSR			EWP	IBOV	CDI
	60	90	120	60	90	120			
Ann. Ret.	**8.21%**	8.19%	7.89%	3.87%	4.82%	3.51%	1.23%	7.49%	9.22%
Cum. Ret.	**71.85%**	71.73%	68.42%	29.75%	38.17%	26.76%	8.77%	64.15%	83.16%
Ann. Vol.	13.92%	13.31%	**12.54%**	15.19%	16.65%	13.76%	16.97%	22.36%	0.16%
Avg. daily $\text{VaR}_{5\%}$	-1.40%	-1.34%	-1.26%	-1.56%	-1.70%	-1.41%	-1.74%	-2.28%	-
Term. Value	**R$ 1.72**	R$ 1.71	R$ 1.68	R$ 1.30	R$ 1.38	R$ 1.27	R$ 1.09	R$ 1.64	R$ 1.83
SR	-6.65%	-7.04%	-9.70%	-32.27%	-24.18%	-37.97%	-43.09%	-7.09%	-
MI	**7.59%**	7.50%	6.85%	1.34%	3.31%	-0.05%	-1.30%	-	-
MD	37.15%	36.48%	33.39%	36.90%	42.24%	**32.93%**	59.81%	45.30%	-
IBOV corr.	1.52%	1.24%	0.79%	0.49%	1.56%	1.11%	0.64%	-	-
CAPM β	0.0095	0.0074	0.0045	0.0031	0.0115	0.0065	0.0045	-	-
CAPM α	0.0000	0.0000	0.0000	-0.0002	-0.0001	-0.0002	-0.0002	-	-
Avg. Turnover	3.24%	4.38%	4.77%	11.66%	13.26%	31.35%	-	-	-
Avg. No. Assets	22	32	38	12	55	19	-	-	-

Figure 6 illustrates the updated value of investing \$ 1 in each portfolio through the out-of-sample period. MVP_{60} and MSR_{90} showed the best performance (MI index) against their counterparts when optimizing weights under an universe of the 60 and 90 most efficient equities. When weights portfolios are updated monthly, the temporal evolution of their cumulative returns showed similarities between the minimum variance and maximum Sharpe ratio strategies, both generally higher than the IBOVESPA. After the middle of 2017, MVP outperforms MSR and showed a quite similar evolution as the IBOVESPA.

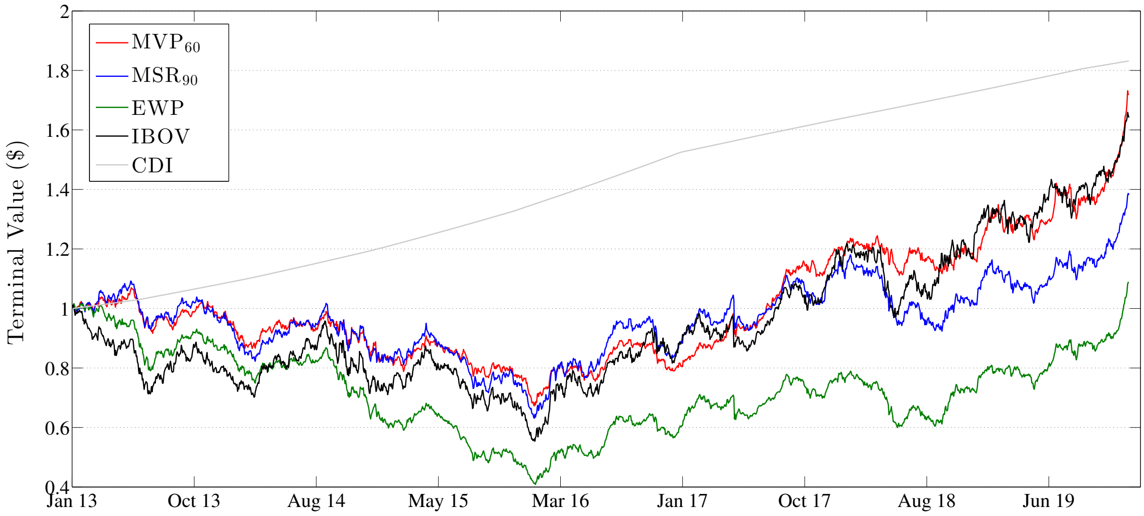


Figure 6 – Terminal value curves of investing \$ 1 in each portfolio, an illustration of cumulative portfolios returns. MVP and MSR curves correspond to the strategies associated with the higher adjusted risk-return relation, measured by the Modigliani index, i.e. considering the 60 and 90 most efficient equities as an universe to trade in, respectively. MVP and MSR were yearly rebalanced.

4.7 Efficient-assets portfolios during COVID-19 pandemic

On December 8th, 2019, a novel Corona Virus Disease (COVID-19), a member of the family of the Severe Acute Respiratory Syndrome Corona-virus-2 (SARS-CoV-2) started to infect people in the city of Wuhan, China (LAI et al., 2020). COVID-19 was declared as pandemic by World Health Organization on March, 11th, 2020 and since then it invaded almost all countries in the world[11].

[11] World Health Organization: <https://www.who.int/health-topics/coronavirus#tab=tab_1>. Access on Aug. 15, 2020.

Nowadays, the COVID-19 has spread fear and anxiety amongst people and investors. This psychological state may therefore lead to behavioural biases like the herding behaviour. In addition, many stock markets around the world, Brazil included, showed significant negative responses to the pandemic. Hence, we also evaluated in this paper the performance of portfolios comprised by the most efficient stocks in an environment of high levels of risk, as during the COVID-19 pandemic.

The effect of the recent COVID-19 pandemic on financial markets has not been deeply developed because of its recentness and uncertainty. This study is therefore an attempt to contribute to the literature by exploring investment strategies which are less prone to the negative effects of the recent coronavirus.

For this analysis, the data were divided again into in-sample and out-of-sample sets. The in-sample data are comprised by the last three years of daily price returns before the year of 2020, i.e. from Jan. 3, 2017 to Dec. 30, 2019 - 738 daily observations. This sample is used to perform the ranking of equities in terms of efficiency, using MF-DFA, as well as for estimating portfolios return and covariance matrix to determine the corresponding weights. The performance of all portfolios is evaluated during the out-of-sample set, which comprises the period from Jan. 2, 2020 to Jul. 17, 2020[12].

Figure 7 presents the histogram of the MDM values calculated for the stocks negotiated at B3 using returns for the new in-sample set: Jan. 3, 2017 to Dec. 30, 2019. Notice again that the levels of efficiency are considerably different among the stocks traded at B3, and the right tail side of the distribution reveals a significant presence of less efficient equities in terms of the weak-form of market efficiency. However, by comparing these values with the MDM distribution based on data from Jan. 4, 2010 to Dec. 28, 2012, as shown in Figure 2, with an average MDM of 0.1817, it is verified that, on a more recent period (Jan. 3, 2017 to Dec. 30, 2019), stocks traded at B3 are less efficient, as the average MDM is now 0.2089. It confirms the literature findings that assets efficiency changes over time (AL-YAHYAEE et al., 2020; TIWARI; AYE; GUPTA, 2019; MENSI et al., 2018; ARSHAD; RIZVI; GHANI, 2016; ALI et al., 2018).

[12] We use the day of Jul. 17, 2020 as ending point as it was the last available datum when the corresponding analyses were performed.

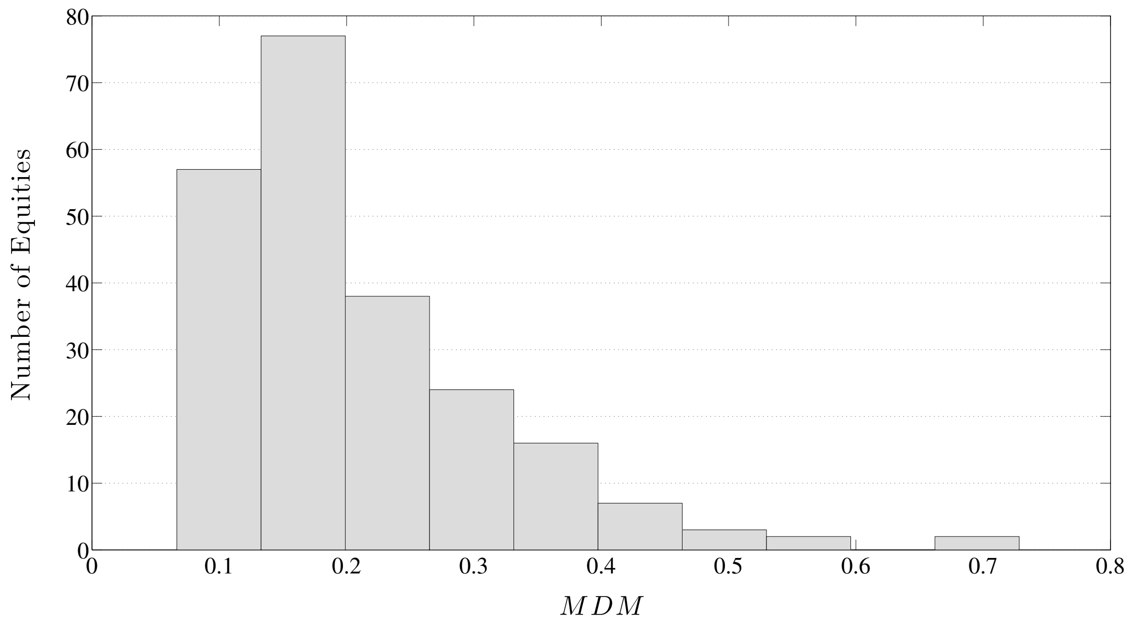


Figure 7 – Histogram of the market deficiency measure (MDM) calculated for the equities traded at B3 using daily price returns for the period from Jan. 3, 2017 to Dec. 30, 2019.

Based on the MDM measure, stocks were ranked in terms of efficiency to compose the corresponding portfolios to be evaluated during the period of the COVID-19 pandemic. Again, Minimum Variance (MVP) and Maximum Sharpe Ratio (MSR) portfolios were optimized in three different cases: concerning an universe of available assets to trade in by the most 60, 90 and 120 efficient stocks. For this analysis, only a non-rebalancing approach is considered, as the aim of the evaluation is to verify whether or not a portfolio with the most efficient stocks are less prone to the negative impacts of a period of high levels of systemic risk, as during the COVID-19 pandemic. Thus, portfolios were composed based on the in-sample data and the obtained weights are kept the same during the remaining out-of-sample period, i.e. from Jan. 2, 2020 to Jul. 17, 2020.

Table 6 provides the performance evaluation of each portfolio concerning different metrics of risk and return, as well as the number of equities in each portfolio (No. Assets) and the corresponding average weights (Avg. Allocation). First, all portfolios showed a considerable impact of the pandemic, as verified by the negative values of annualized return and cumulative return measures. However, the Brazilian stock market benchmark, the IBOVESPA index, and the equally weighted portfolio (EWP), are the portfolios corresponding to the lowest values of cumulative returns, revealing the huge impact of the pandemic in the Brazilian equity market. It is worth notice that, during march 2020, the B3 stock exchange suffered six circuit breakers, which occur when is verified an oscillation of -10% from IBOVESPA's closing level on the previous day. In such cases, all trading in equities segment is interrupted for 30 minutes and during this period, the orders that are not participating in auction can be cancelled.

In terms of returns, results from Table 6 showed that minimum variance and maximum Sharpe ratio portfolios provided higher returns (lower losses) than IBOVESPA and EWP, independently when a different number of efficient stocks are available to trade in and to optimize the weights. MVP portfolio, when the most efficient 90 stocks are used to obtain the weights, showed a level of losses that is approximately 40% smaller than the IBOVESPA, in terms of annualized returns. It means that, for this case, besides MVP approach showed a negative impact of the COVID-19 crisis, the losses are considerably smaller than the IBOVESPA. In addition, MSR, concerning the most efficient 120 stocks, showed a cumulative return of -10.22%, which compared to IBOVESPA, with a cumulative return of -19.37%, provides an approach that losses are approximately 47% lower than the observed by the Brazilian main stock index.

When we move to the evaluation of portfolio's risk, results from Table 6 show that MVP, in all cases, provided the lowest annualized volatility and average daily VaR. Annualized volatility of IBOVESPA, EWP and MSR are on average 55%, whereas MVP portfolios showed an average value of 37%, which corresponds in a lower level of risk of approximately 33%. By considering simultaneously risk and return, by analysing Sharpe ratio (SR) and Modigliani index (MI) measures, it is noticed from Table 6 that these metrics are consistently negative, as portfolios negative returns are lower than the risk-free interest for the period, measured by the CDI, being the corresponding interpretations problematic. In addition, the maximum drawdown (MD) values are higher for MSR, EWP and IBOVESPA.

Table 6 – Portfolios performance metrics with non-rebalancing during COVID-19 pandemic. The results are calculated for the out-of-sample set, from the period from Jan. 2, 2020 to Jul. 17, 2020. MVP, MSR and EWP stand for the minimum variance, maximum Sharpe ratio and equally weighted portfolios. IBOV is the IBOVESPA index and CDI the risk-free interest rate. MVP and MSR are optimized considering the most 60, 90 and 120 efficient stocks, ranked using MF-DFA for the returns data from the in-sample set, from Jan. 3, 2017 to Dec. 30, 2019. Term. Value is the terminal value of investing \$ 1 in each portfolio during the period evaluated. No. Assets is the number of assets in each portfolio. Avg. Allocation is the average portfolio weights.

Strategies	MVP			MSR			EWP	IBOV	CDI
	60	90	120	60	90	120			
Ann. Ret.	-19.97%	-19.68%	-22.63%	-26.27%	-20.52%	**-18.10%**	-35.32%	-32.90%	2.44%
Cum. Ret.	-11.33%	-11.16%	-12.93%	-15.17%	-11.66%	**-10.22%**	-20.95%	-19.37%	1.31%
Ann. Vol.	38.85%	**37.44%**	37.58%	53.50%	55.52%	52.02%	56.77%	58.65%	0.03%
Avg. daily $\text{VaR}_{5\%}$	-4.22%	-4.07%	-4.06%	-5.79%	-5.98%	-5.60%	-6.20%	-6.36%	-
Term. Value	R\$ 0.89	R\$ 0.89	R\$ 0.87	R\$ 0.85	R\$ 0.88	**R\$ 0.90**	R\$ 0.79	R\$ 0.81	R\$ 1.01
SR	-56.34%	-57.71%	-65.84%	-52.40%	-40.39%	-38.56%	-64.93%	-58.83%	-
MI	-31.40%	-32.22%	-37.11%	-29.04%	-21.82%	-20.72%	-36.56%	-	-
MD	**37.98%**	38.06%	39.59%	48.40%	51.13%	49.27%	55.41%	50.35%	-
IBOV corr.	87.82%	87.50%	88.26%	91.47%	90.20%	89.82%	92.40%	-	-
CAPM β	0.5818	0.5587	0.5599	0.8344	0.8536	0.7964	0.8947	-	-
CAPM α	-0.0001	-0.0001	-0.0003	0.0001	0.0005	0.0004	-0.0003	-	-
No. Assets	20	24	27	15	19	23		-	-
Avg. Allocation	1.67%	1.11%	0.92%	2.64%	1.78%	1.93%		-	-

All MVP and MSR portfolios do provide a higher correlation to the IBOVESPA index, in comparison to the analyses in the previous subsections, which is expected in a scenario of higher levels of systemic risk as during the COVID-19 pandemic (see Table 6). This is also verified by moving the analysis to the corresponding CAPM betas, that are generally higher for MVP, MSR and EWP. As CAPM beta is associated as a measure of portfolios risk, the systemic risk related to the market benchmark, the IBOVESPA, it is verified that portfolios composed by the most efficient stocks produced lower systemic risks than the IBOVESPA, as the betas are lower than the unit. Finally, MVP invested in an average of 24 stocks, with a corresponding allocation of 1.23% in each equity (Table 6).

Figure 8 shows the evolution of portfolios cumulative returns, illustrated by the terminal value temporal behavior of investing $ 1 in each strategy at the beginning of the out-of-sample period (the COVID-19 pandemic). The curves related to MVP and MSR concern the cases associated with higher returns and lower volatilities (see Table 6), i.e. using the most 90 and 120 efficient stocks to compute portfolios weights, respectively. At the end of Feb. 2020, all portfolios cumulative returns suffered a considerable decrease, which is consistently during Mar. 2020, the month related to the six circuit breakers occurred at B3, as a market response to the increasing number of COVID-19 infections in Brazil. At the end of Mar. 2020, cumulative returns showed a recovery as the the uncertainties related to the coronavirus reduced, as well as the emergence of news associated to the development of a potential vaccine.

Generally, the analysis concerning the period of the COVID-19 pandemic showed that portfolios composed by the most efficient equities negotiated at B3, for both minimum variance and maximum Sharpe ratio strategies, suffered the negative consequences of the recent crisis, as also verified by the Brazilian stock market benchmark, the IBOVESPA, as expected in a scenario of high systemic risk. However, the stock efficient-based portfolios resulted in losses that are significantly lower then IBOVESPA and EWP portfolios, indicating that the suggest approach is able to produce more efficient portfolios, associated with higher returns with lower levels of risk, in line with the previous analyses for the period excluding the recent COVID-19 pandemic.

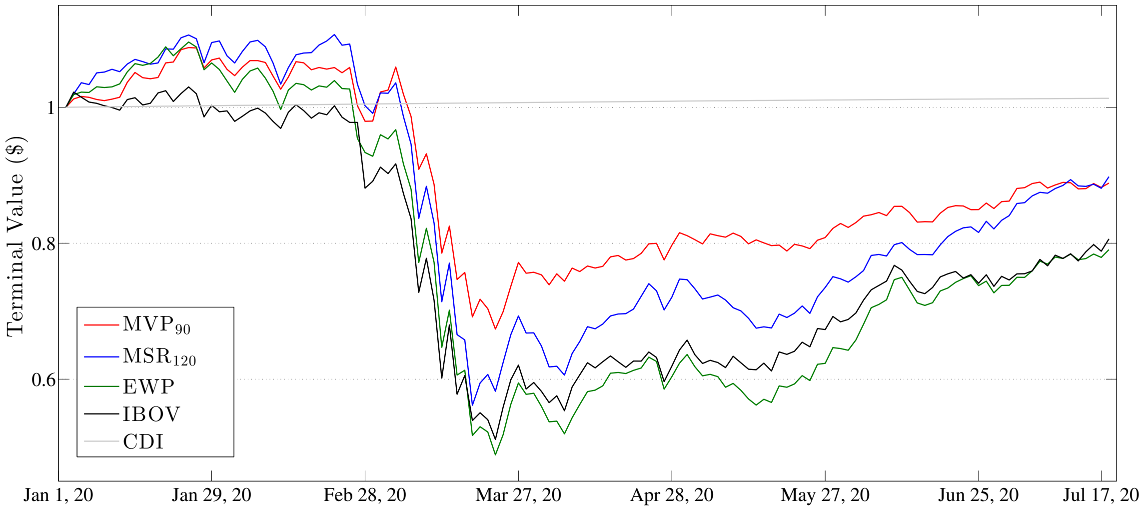


Figure 8 – Terminal value curves of investing $ 1 in each portfolio, as an illustration of cumulative portfolios returns during the COVID-19 pandemic crisis - from Jan. 2, 2020 to Jul. 17, 2020. MVP and MSR curves correspond to the strategies associated with the higher adjusted risk-return relation, measured by the annualized returns and volatilities measures, i.e. considering the 60 and 120 most efficient equities as an universe to trade in, respectively.

4.8 Main empirical findings

Based on the empirical experiments reported, under different scenarios for portfolios weights rebalancing, the main findings of this work can be summarized as follows:

- multifractal detrendend fluctuation analysis (MF-DFA) indicated that the equities traded at B3 showed different levels of efficiency, in terms adherence to the weak-form of the efficient market hypothesis. Generally, the stocks showed a multifractality nature with an evidence of long-term persistence for the corresponding price returns;
- by performing a trading strategy that considered only the stocks with higher levels of efficiency to optimize portfolios weights, when a minimum variance approach is taken into account, significantly better performance was achieved, with higher levels of risk-adjusted returns, when compared against the main stock market benchmark in Brazil, the IBOVESPA index;
- during the period considered, a Maximum Sharpe ratio selecting portfolio strategy was not able to produce higher returns than MVP when both were optimized under an universe of the most efficiency equities;

- the approach suggested in this work indicates that composing portfolios using only the most efficient stocks generates returns associated with lower systemic risk, as their corresponding CAPMs betas showed, for all cases, values significantly lower then the unit. This behavior was reflected in the temporal evolution of portfolios cumulative returns, as the resulting stock efficiency-based approach was related to a less variation (drawdowns) due to changes in the stock market, represented by the IBOVESPA;

- minimum variance portfolios consistently produced higher risk-adjusted returns independently on the number of efficient stocks considered to trade in and to determine the respective weights, being a simple and replicable strategy associated with the lowest levels of risk. When the rebalancing frequency rises, it negatively affects the MVP performance, which may be due to the changes on the level of efficiency of the corresponding stocks, as efficiency is time-varying;

- MVP provides a performance that is less sensitive to the frequency of rebalancing in comparison to the MSR portfolio. Also, the minimum variance approach reached the best performance investing on 30 stocks on average, with a lower level of average turnover, implying reduced transaction costs;

- during the Corona Virus Disease (COVID-19) pandemic, our experiments have showed that portfolios composed by the most efficient equities traded at B3 are less prone to the negative consequents of the recent systemic crisis, comprising an alternative approach for periods of high market instabilities.

Summing up, the empirical results provided evidences of an alternative and potential trading approach to market participants in the Brazilian stock market, with a minimum risk portfolio optimized considering only the most efficient equities. This strategy was able to consistently produce higher levels of adjusted-risk returns than the main stock benchmark, the IBOVESPA, even in periods of high systemic risk, as during the COVID-19 pandemic.

5 Conclusion

This study proposed a new trading strategy in the Brazilian equity market. From econophysics, multifractal detrended fluctuation analysis (MF-DFA) was considered to measure the level of efficiency of the equities traded at B3. MF-DFA provides a calculation of a market deficiency measure that is used to compute the degree of financial market efficiency and ranking the corresponding stocks. Thus, it was suggested a trading strategy for portfolio selection based on the most efficient assets under the mean-variance framework of Markowitz with two different approaches: the minimum variance (MVP) and the maximum Sharpe ratio (MSR) portfolios. Using data from all equities traded at the B3 during the period from January 2010 to July 2020, portfolios performance were evaluated concerning several risk-return metrics with different rebalancing frequencies. The suggested methodology was also compared against the main Brazilian stock benchmark, the IBOVESPA index, an equally weighted portfolio and the risk-free interest rate in Brazil, measure by the interbank loans rate, the CDI. Additionally, experiments were also particularly conducted to evaluated the portfolios performance during the COVID-19 pandemic, in order to provide empirical evidence of the suggested approach potential for periods of considerable market instabilities.

MF-DFA analysis indicated that a multifractal nature for asset price returns was confirmed in the Brazilian stock market, where the equities showed different levels of efficiency, generally associated with long-term persistence. The MVP portfolio produces significantly higher performance than the alternative competitors, both concerning risk and return performance metrics. The strategy for optimizing portfolios weights under the consideration of only the most efficient equities provides lower levels of systemic risk (CAPM betas), indicating that the lack of efficiency is related to higher sensitivity to macroeconomic and conjuncture changes in the economy. Additionally, the minimum risk approach showed consistently higher returns than the IBOVESPA, with a level of risk generally twice smaller than this benchmark. Finally, concerning the analysis during the COVID-19 pandemic, besides the consistent negative impact of the crisis in all portfolios performance, portfolios composed by the most efficient stocks verified significantly better results (lower losses) than the IBOVESPA.

The empirical results of this paper have several implications for academics and market practitioners. From a theoretical point of view, it was verified that the efficient market hypothesis, used as a prerogative for several financial models, does not hold for the Brazilian equity market, as all equities showed a multifractality nature and long-range dependences for their price returns dynamics. Further, the minimum variance portfolio presented performance associated with higher returns and lower risks when compared against the competitors, which is not in accordance to the trade-off of a positive relation with risk and return, stated by the modern financial theory, indicating the presence of microstructure noises and arbitrage possibilities. For practitioners, the findings reported in this research provides the suggestion of a simple and profitable trading strategy in the Brazilian equity market, which was able to overperform the main stock benchmark in Brazil, the IBOVESPA, with higher returns and significantly lower risk. Additionally, results also indicated that accessing stocks level of efficiency plays an important role in portfolio management, even in periods of crisis, as during the COVID-19 pandemic. This work can be extended in several manners, for instance: including transaction costs in the analysis, the possibility of investing jointly on a risk-free interest rate, and the inclusion of short positions; use of more sophisticated approaches to the covariance matrix estimation; and the combination of machine learning prediction models with portfolio optimization, as the evidence of long-range dependencies on assets returns.

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Appendix A

Table A.1 – Ranking of equities traded at the B3 in terms of a market deficiency measure (MDM), calculated from MF-DFA analysis. Lower values of MDM indicates a higher level of market efficiency. For each stock, it is reported their corresponding ranking on the sample considered, company name, and the associated main economic activity according to B3 classification.

Rank	B3 Ticker	Company Name	Economic Activity	MDM
1	CPFE3	CPFL ENERGIA	Electric	0.0815
2	CPLE6	CIA PARANAENSE DE ENERGIA	Electric	0.0889
3	SANB3	BCO SANTANDER	Banks	0.0975
4	ENGIE	ENGIE BRASIL ENERGIA	Electric	0.0987
5	GOAU4	METALURGICA GERDAU	Steel and Metalurgy	0.1003
6	LAME4	LOJAS AMERICANAS	Diversified Retailers	0.1012
7	JBSS3	JBS	Meat. Poultry and Others	0.1026
8	BRAP4	BRADESPAR	Metalic Minerals	0.1054
9	BRAP3	BRADESPAR	Metalic Minerals	0.1074
10	KLBN4	KLABIN	Pulp and Paper	0.1076
11	TPIS3	TRIUNFO PARTICIP. E INVEST.	Toll Roads and Highways	0.1097
12	SBSP3	CIA SANEAMENTO BASICO EST SAO PAULO	Water Utilities	0.1100
13	TOTS3	TOTVS	Software and Services	0.1116
14	B3SA3	BRASIL. BOLSA. BALCÃO	Diversified Financial Services	0.1122
15	LREN3	LOJAS RENNER	Apparel. Fabric and Footwear	0.1124
16	ITSA4	ITAUSA INVESTIMENTOS ITAU	Banks	0.1135
17	BBAS3	BCO BRASIL	Banks	0.1136
18	BRFS3	BRF	Meat. Poultry and Others	0.1140
19	BEEF3	MINERVA	Meat. Poultry and Others	0.1150
20	MULT3	MULTIPLAN - EMPREEND IMOBILIARIOS	Real State	0.1150
21	CCRO3	CCR	Toll Roads and Highways	0.1154
22	FESA4	CIA FERRO LIGAS DA BAHIA	Steel	0.1171
23	LIGT3	LIGHT	Electric	0.1173
24	POMO3	MARCOPOLO	Transportation Equipment and Components	0.1179
25	GUAR3	GUARARAPES CONFECCOES	Apparel. Fabric and Footwear	0.1186
26	VIVT3	TELEFÔNICA BRASIL	Telecommunications	0.1197
27	ITUB4	ITAU UNIBANCO HOLDING	Banks	0.1199
28	GGBR3	GERDAU	Steel and Metalurgy	0.1203
29	BBDC4	BCO BRADESCO	Banks	0.1206
30	GGBR4	GERDAU	Steel and Metalurgy	0.1206
31	BBRK3	BRASIL BROKERS PARTICIPACOES	Property Agency	0.1215
32	RENT3	LOCALIZA RENT A CAR	Car Rental	0.1221
33	BRSR6	BCO ESTADO DO RIO GRANDE DO SUL	Banks	0.1222
34	RSID3	ROSSI RESIDENCIAL	Real State	0.1234
35	BBDC3	BCO BRADESCO	Banks	0.1242

Table A.1 Continued. – Ranking of equities traded at the B3 in terms of a market deficiency measure (MDM), calculated from MF-DFA analysis. Lower values of MDM indicates a higher level of market efficiency. For each stock, it is reported their corresponding ranking on the sample considered, company name, and the associated main economic activity according to B3 classification.

Rank	B3 Ticker	Company Name	Economic Activity	MDM
36	NTCO3	NATURA & CO HOLDING	Personal Care and Cleaning Products	0.1249
37	MRVE3	MRV ENGENHARIA E PARTICIPACOES	Real State	0.1254
38	UNIP6	UNIPAR CARBOCLORO	Chemicals	0.1266
39	WSON33	WILSON SONS	Warehousing and Storage	0.1270
40	IGTA3	IGUATEMI EMPRESA DE SHOPPING CENTERS	Real State	0.1275
41	POMO4	MARCOPOLO	Transportation Equipment and Components	0.1284
42	MYPK3	IOCHPE MAXION	Automobiles and Motorcycles	0.1286
43	AMAR3	MARISA LOJAS	Apparel. Fabric and Footwear	0.1290
44	VALE3	VALE	Metalic Minerals	0.1290
45	SLCE3	SLC AGRICOLA	Agriculture	0.1294
46	HYPE3	HYPERA	Pharmaceutical	0.1298
47	EUCA4	EUCATEX	Wood and Paper	0.1302
48	PETR3	PETROLEO BRASILEIRO PETROBRAS	Oil, Gas and Biofuels	0.1303
49	ITUB3	ITAU UNIBANCO HOLDING	Banks	0.1322
50	ABCB4	BCO ABC BRASIL	Banks	0.1331
51	ABEV3	AMBEV	Beer and Soft Drinks	0.1337
52	BRML3	BR MALLS PARTICIPACOES	Real State	0.1389
53	ENEV3	ENEVA	Electric Utilities	0.1406
54	GRND3	GRENDENE	Footwear	0.1407
55	PETR4	PETROLEO BRASILEIRO PETROBRAS	Oil, Gas and Biofuels	0.1408
56	WHRL4	WHIRLPOOL	Household Appliance	0.1423
57	ROMI3	INDUSTRIAS ROMI	Machines and Industrial Equipments	0.1429
58	GOAU3	METALURGICA GERDAU	Steel and Metalurgy	0.1441
59	TCSA3	TECNISA	Real State	0.1442
60	ENBR3	EDP - ENERGIAS DO BRASIL	Electric Utilities	0.1446
61	CYRE3	CYRELA BRAZIL REALTY S.A.EMPREEND E PART	Real State	0.1452
62	SMTO3	SAO MARTINHO	ugar - Alcohol	0.1458
63	USIM3	USINAS SID DE MINAS GERAIS	Steel	0.1460
64	WEGE3	WEG	Motors and Compressors	0.1468
65	SLED4	SARAIVA LIVREIROS	Diversified Retailers	0.1473
66	TIMP3	TIM PARTICIPACOES	Telecommunications	0.1482
67	RAPT4	RANDON	Transportation Equipment and Components	0.1494
68	CPLE3	CIA PARANAENSE DE ENERGIA	Electric Utilities	0.1514
69	VIVR3	VIVER INCORPORADORA E CONSTRUTORA	Real State	0.1517
70	YDUQ3	YDUQS PARTICIPACOES	Education Services	0.1525
71	CSAN3	COSAN	Oil, Gas and Biofuels	0.1548
72	SANB11	BCO SANTANDER	Banks	0.1567

Table A.1 Continued. – Ranking of equities traded at the B3 in terms of a market deficiency measure (MDM), calculated from MF-DFA analysis. Lower values of MDM indicates a higher level of market efficiency. For each stock, it is reported their corresponding ranking on the sample considered, company name, and the associated main economic activity according to B3 classification.

Rank	B3 Ticker	Company Name	Economic Activity	MDM
73	LLIS3	RESTOQUE COMÉRCIO E CONFECCOES DE ROUPAS	Apparel. Fabric and Footwear	0.1569
74	EVEN3	EVEN CONSTRUTORA E INCORPORADORA	Real State	0.1579
75	SGPS3	SPRINGS GLOBAL PARTICIPACOES	Textiles. Apparel and Footwear	0.1585
76	DTEX3	DURATEX	Wood and Paper	0.1596
77	HGTX3	CIA HERING	Textiles. Apparel and Footwear	0.1636
78	CRDE3	CR2 EMPREENDIMENTOS IMOBILIARIOS	Real State	0.1638
79	EMBR3	EMBRAER	Aerospace and Defense	0.1641
80	EQTL3	EQUATORIAL ENERGIA	Electric Utilities	0.1664
81	ITSA3	ITAUSA INVESTIMENTOS ITAU	Banks	0.1669
82	DIRR3	DIRECIONAL ENGENHARIA	Real State	0.1677
83	MMXM3	MMX MINERACAO E METALICOS	Metalic Minerals	0.1704
84	RDNI3	RNI NEGÓCIOS IMOBILIÁRIOS	Real State	0.1713
85	FRTA3	POMIFRUTAS	Agriculture	0.1725
86	ALPA4	ALPARGATAS	Textiles. Apparel and Footwear	0.1735
87	MDIA3	M.DIAS BRANCO S.A. IND COM DE ALIMENTOS	Food Processors	0.1759
88	LAME3	LOJAS AMERICANAS	Diversified Retailers	0.1785
89	VIVT4	TELEFÔNICA BRASIL	Telecommunications	0.1799
90	BTOW3	B2W - COMPANHIA DIGITAL	Diversified Retailers	0.1816
91	VLID3	VALID SOLUCOES	Diversified Services	0.1837
92	TRIS3	TRISUL	Real State	0.1837
93	RADL3	RAIA DROGASIL	Pharmaceutical	0.1841
94	JHSF3	JHSF PARTICIPACOES	Real State	0.1858
95	PDGR3	PDG REALTY S.A. EMPREEND E PARTICIPACOES	Real State	0.1861
96	SCAR3	SAO CARLOS EMPREEND E PARTICIPACOES	Real State	0.1862
97	PSSA3	PORTO SEGURO	Insurance	0.1874
98	SANB4	BCO SANTANDER	Banks	0.1883
99	SULA11	SUL AMERICA	Insurance	0.1889
100	CSMG3	CIA SANEAMENTO DE MINAS GERAIS-COPASA	Water Utilities	0.1891
101	POSI3	POSITIVO TECNOLOGIA	Hardware and Equipments	0.1903
102	USIM5	USINAS SID DE MINAS GERAIS	Steel and Metalurgy	0.1904
103	IDNT3	IDEIASNET	Diversified Financial Services	0.1916
104	TGMA3	TEGMA GESTAO LOGISTICA	Transportation	0.1916
105	GPIV33	GP INVESTMENTS	Diversified Financial Services	0.1919
106	OIBR4	OI	Telecommunications	0.1922
107	COCE5	CIA ENERGETICA DO CEARA	Electric Utilities	0.1936

Table A.1 Continued. – Ranking of equities traded at the B3 in terms of a market deficiency measure (MDM), calculated from MF-DFA analysis. Lower values of MDM indicates a higher level of market efficiency. For each stock, it is reported their corresponding ranking on the sample considered, company name, and the associated main economic activity according to B3 classification.

Rank	B3 Ticker	Company Name	Economic Activity	MDM
108	GSHP3	GENERAL SHOPPING E OUTLETS DO BRASIL	Real State	0.1949
109	CIEL3	CIELO	Diversified Financial Services	0.1953
110	ODPV3	ODONTOPREV	Hosp Serv. Anal and Diagnostics	0.1953
111	TUPY3	TUPY	Transportation Equipment and Components	0.1959
112	HBOR3	HELBOR EMPREENDIMENTOS	Real State	0.1971
113	RCSL4	RECRUSUL	Transportation Equipment and Components	0.1978
114	CARD3	CSU CARDSYSTEM	Diversified Services	0.1979
115	GFSA3	GAFISA	Real State	0.1983
116	LPSB3	LPS BRASIL - CONSULTORIA DE IMOVEIS	Property Agency	0.2005
117	FLRY3	FLEURY	Hosp Serv. Anal and Diagnostics	0.2033
118	TCNO4	TECNOSOLO ENGENHARIA	Construction and Engeneering	0.2076
119	GOLL4	GOL LINHAS AEREAS INTELIGENTES	Airlines	0.2130
120	BRKM5	BRASKEM	Petrochemicals	0.2134
121	EZTC3	EZ TEC EMPREEND. E PARTICIPACOES	Real State	0.2215
122	IDVL4	BCO INDUSVAL	Banks	0.2233
123	BRKM3	BRASKEM	Petrochemicals	0.2249
124	AGRO3	BRASILAGRO - CIA BRAS DE PROP AGRICOLAS	Agriculture	0.2274
125	TCNO3	TECNOSOLO ENGENHARIA	Construction and Engeneering	0.2279
126	TASA4	TAURUS ARMAS	Weapons and Munitions	0.2294
127	FHER3	FERTILIZANTES HERINGER	Fertilizers	0.2300
128	CTKA4	KARSTEN	Textiles. Apparel and Footwear	0.2302
129	CMIG4	CIA ENERGETICA DE MINAS GERAIS	Electric Utilities	0.2311
130	CSNA3	CIA SIDERURGICA NACIONAL	Steel and Metalurgy	0.2340
131	TESA3	TERRA SANTA AGRO	Agriculture	0.2406
132	CCPR3	CYRELA COMMERCIAL PROPERT	Real State	0.2407
133	CGAS5	CIA GAS DE SAO PAULO	Gas Utilities	0.2411
134	OIBR3	OI	Telecommunications	0.2493
135	MRFG3	MARFRIG GLOBAL FOODS	Meat. Poultry and Others	0.2498
136	SAPR4	CIA SANEAMENTO DO PARANA	Water Utilities	0.2536
137	PMAM3	PARANAPANEMA	Copper Products	0.2599
138	CMIG3	CIA ENERGETICA DE MINAS GERAIS	Electric Utilities	0.2626
139	BAZA3	BCO AMAZONIA	Banks	0.2662
140	ETER3	ETERNIT	Building Products	0.2665
141	CLSC4	CENTRAIS ELET DE SANTA CATARINA	Electric Utilities	0.2673

Table A.1 Continued. – Ranking of equities traded at the B3 in terms of a market deficiency measure (MDM), calculated from MF-DFA analysis. Lower values of MDM indicates a higher level of market efficiency. For each stock, it is reported their corresponding ranking on the sample considered, company name, and the associated main economic activity according to B3 classification.

Rank	B3 Ticker	Company Name	Economic Activity	MDM
142	LUPA3	LUPATECH	Oil, Gas and Biofuels	0.2679
143	PINE4	BCO PINE	Banks	0.2727
144	PTBL3	PBG	Building Products	0.2765
145	LOGN3	LOG-IN LOGISTICA INTERMODAL	Marine and Water Transport	0.2835
146	BPAN4	BCO PAN	Banks	0.2914
147	KEPL3	KEPLER WEBER	Machines and Industrial Equipments	0.2947
148	PFRM3	PROFARMA DISTRIB PROD FARMACEUTICOS	Pharmaceutical and Others Products	0.3036
149	BEES3	BANESTES	Banks	0.3221
150	ELET3	CENTRAIS ELET BRAS	Electric Utilities	0.3224
151	GPCP3	GPC PARTICIPACOES	Petrochemicals	0.3412
152	TELB4	TELEC BRASILEIRAS	Telecommunications	0.3525
153	CESP6	CESP - CIA ENERGETICA DE SAO PAULO	Electric Utilities	0.3620
154	TRPL4	CETTEP	Electric Utilities	0.3676
155	ELET6	CENTRAIS ELET BRAS	Electric Utilities	0.3704
156	IGBR3	IGB ELETRÔNICA	Real State	0.3995
157	CESP3	CESP - CIA ENERGETICA DE SAO PAULO	Electric Utilities	0.4164
158	CGAS3	CIA GAS DE SAO PAULO	Gas Utilities	0.4833